Triad Trade and Small Worlds of Large Spatial Production Networks

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Abstract

Do firms' production network contacts (suppliers, customers) help form connections with newer suppliers or customers? If they did, one would observe transitive triads – a group of three firms all trading with each other - in production networks. Using rich administrative data on firm-to-firm linkages from India, I provide first evidence that triads are excessively prevalent in firm-to-firm production networks. Further, I find that proximity through the production network is an important determinant of trade frictions. I develop a quantitative general equilibrium model of network formation between spatially distant firms. The model lends to elegant aggregation and features endogenous trade frictions unlike standard trade models. The estimated model implies that network proximity explains a dominant majority of trade frictions.

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1 Introduction

Trade frictions play a key role in models of international trade and economic geography that seek to explain gains from market integration and unequal distribution of economic activity. A large part of why countries or regions do not trade with each other as much as a simple model predicts is attributed to trade frictions. Yet these frictions are traditionally assumed to be of the iceberg form and largely attributed to distance, travel times and other such measures of geographic proximity. Firm-to-firm trade in large spatial production networks underlies much of trade across space. These networks often span across borders within and across countries, and they can involve a large number of firms. Can the network structure of production across space inform us about such costs?

A production network is made up of firms as nodes and supply chain relationships as links. The smallest unit that consists of both these network elements is a dyad made up of two nodes (a buyer and a supplier) and the link that connects them (a buyer–supplier relationship). Most studies that seek to understand the origins of trade costs across space have focused on dyadic relationships (e.g., buyer–supplier, origin-destination), as all relationships in a network begin with a dyad. In this paper, I focus on triads as units of large production networks. Why the focus on triads? A triad is the smallest network unit where we can observe how a link affects a link or a node affects a link either directly or indirectly connected — network dynamics that a dyad by itself cannot capture. To understand how links that constitute complex networks affect trade costs, we have to begin by studying triads.

In this paper, I utilize new micro-data on firm-to-firm linkages between Indian firms to document excess prevalence of triads in production networks and shine light on new origins of trade frictions through proximity on firm-to-firm production networks. Using data on 103 million firm-to-firm relationships assembled from administrative VAT records spanning across 5 years and pertaining to around 2.5 million Indian firms located across 141 districts, I find that firms are more likely to trade with production network contacts (customers, suppliers) of their production network contacts than other firms. Almost two-thirds of the firm-to-firm connections in the production network are part of transitive triads - a triplet of firms that are all connected to each other.[1] Transitive triads occur 13,750 times more than what a baseline random network formation model would predict. When accounting for spatial heterogeneity, transitive triads occur 600 times more frequently and when accounting for heterogeneity in firm's connectedness, transitive triads occur 90

[1] On the contrary, if there is exactly one pair of firms in a triplet that are not connected, the triad is said to be intransitive.

times more frequently.

Motivated by these facts, I develop a quantitative model of network formation between spatially distant firms. In the model, firms' production processes consist of multiple input requirements. Potential suppliers differ in the suitability of their goods for each of these requirements. Firms randomly encounter potential suppliers and select the most cost-effective suppliers for their production requirements. Firms are more likely to meet a potential supplier if it it is a firm with which they have a common connection. Conditional on meeting, firms are more likely to select a potential supplier for a larger proportion of their requirements if it is able to sell at a lower price and produces a good that is more suitable for its production requirements. The model is tractable even for large numbers of firms and lends itself to elegant aggregation. At the aggregate level, trade flows depend not only on variables that capture geographic proximity but also those that capture network proximity. I find that estimated trade frictions between Indian districts are dominantly explained (75%) by network proximity, measured by second-order connectedness between districts.

In contrast to standard trade models, trade frictions are determined endogenously in the model here. This is because firm-to-firm connections are first formed due to geographic proximity which then leads to formation of further connections due to network proximity and so on. To take endogenous determination of trade frictions into account, I develop a new procedure for counterfactual analysis, modified relative to the exact hat algebra approach. In the exact hat algebra approach (Dekle et al. (2008)) one solves for the fixed point in terms of changes in wages and market access in response to shocks through a tatonnement algorithm. The difference here is that the model leads to an additional fixed point equation for trade frictions which needs to be solved simultaneously. Owing to the non-linear structure of equations that solve for changes in market access, wages and trade frictions simultaneously, the impact of trade costs shocks on trade shares is non-linear unlike the class of models studied in Arkolakis et al. (2012).

Related Literature This paper contributes to three strands of literature.

First, it relates to the literature that studies the role of search and information frictions in trade. Allen (2014), Startz (2021), and Dasgupta and Mondria (2018) attribute it to information frictions that imply that customers and suppliers do not have complete information about available alternatives. Eaton et al. (2016) and Arkolakis et al. (2021) propose models of network formation with search and matching frictions to quantify what proportion of trade costs can be attributed such frictions as opposed to iceberg trade costs. Relative to these papers, I introduce network proximity as a channel for information frictions when firms are looking for suitable suppliers.

Second, it relates to the literature that propose models where new export destinations are similar to a firms' previous destinations. These include Chaney (2014, 2018) and Morales et al. (2019). Relative to these papers, I use observed micro-data on firm-to-firm linkages to construct new measures of network proximity between locations such that similarity between previous and new export destinations arises from observed network proximity.

Finally, it is related to papers that developed model of endogenous production network formation between spatially distant firms. These include Eaton et al. (2016), Panigrahi (2022), and Arkolakis et al. (2021). Relative to these papers, I introduce a model of network formation where trade frictions are endogenously determined via network proximity.

2 Data & Stylized Facts

In this section, I document new facts about firm-to-firm production networks that show that network proximity affects likelihood of firm-to-firm trade. I begin by describing the source of data. I then describe subnetwork configurations that are helpful in inferring how network proximity affects firm-to-firm trade. I follow up with calculations that suggest that such subnetwork configurations appear in much larger numbers when compared to standard statistical models.

2.1 Sources of Data

The primary dataset for this paper consists of the universe of firm-to-firm transactions assembled from commercial tax authorities of five Indian states (viz. Gujarat, Maharashtra, Tamil Nadu, Odisha, and West Bengal) between 2011-12 and 2015-16. These states had a nominal GDP of $738 billion in 2015-16, accounting for nearly 40% of GDP. Among these states, the largest (Maharashtra) accounts for roughly 14% of national GDP while the smallest (Odisha) accounts for a little over 2%. It includes transactions between all firms registered under the value- added tax system in these states. The dataset records 103 million inter- firm relationships between approximately 2.5 million firms located across 141 districts in these 5 states.

2.2 Transitive and Intransitive Triads in Production Networks

I look at subnetwork configurations that help infer whether network proximity affects firm-to-firm trade. I focus on triads as units of large production networks. Why the focus on triads? A triad is the smallest network unit where we can observe how a link affects a link or a node affects a link either directly or indirectly connected. If having a common connection on the production network, makes it more likely for a pair of firms to directly trade with each other, then the pair along with the firm they are both connected to would form a transitive triad. Triads in production networks can be either transitive or intransitive.

A triad is a group of three firms such that there is at least one firm in the group that is connected both of the other two firms. If there is only one such firm, the triad is intransitive. If all firms are such, then the triad is transitive. To understand this terminology, let's consider three firms A, B, and C. Suppose A trades with B and B trades with C. If A does not trade with C, then the triad is intransitive making B the only firm in the group to be connected to both of the other firms, A and C. If A trades with C, then the triad is transitive and all firms in the group are connected to the other two firms.

This classification of triads would be exhaustive for undirected graphs - networks where the relationship between two nodes is not directed. Production networks are directed graphs - networks where the relationship between two nodes (firms) is directed. That is, if A sells intermediate inputs to B, then B buys intermediate inputs from A. Hence, the classification of triads, both transitive and intransitive, is richer. On one hand, intransitive triads occur in three unique configurations. First, they can appear as a divergent intransitive triad - when A buys from B and C buys from B. Second, they can appear as a convergent intransitive triad - when A sells to B and C sells to B. Finally, they can appear as a chain intransitive triad - when A sells to B and B sells to C. On the other hand, transitive triads occur in two unique configurations: cyclic and acyclic transitive triads. A transitive triad would be acyclic if A sells to B, B sells to C, and A also sells to C. A transitive triad would be cyclic if A sells to B, B sells to C, and C in turn sells to A. Figure 2.1 depicts various configurations of transitive and intransitive triads.

To count the number of different triads in the production network, I utilize the adjacency matrix associated with the production network. Let A denote the adjacency matrix of the production network where the $(i, j)^{th}$ element takes the value 1 if i is a supplier to j and 0 if it is not. Let $\mathbf{1}$ denote a vector of ones. Table 1 provides formulas for computing counts of different configurations from network data. The number of intransitive and transitive triads in the production network are reported in Table 2. It shows that among

Figure 2.1: Intransitive and Transitive Triad Configurations

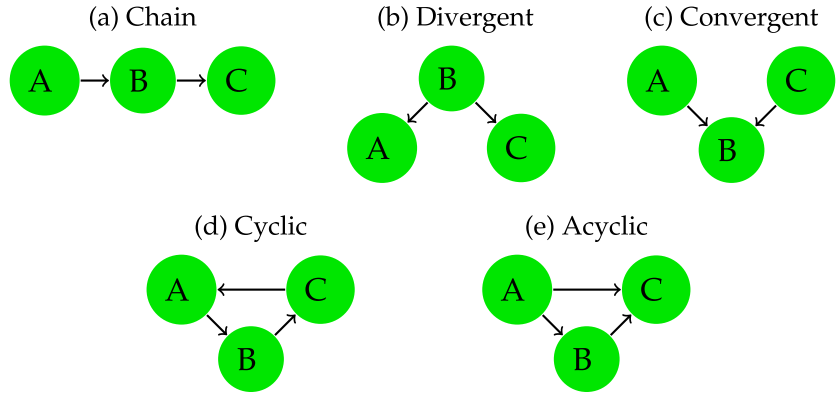


Table 1: Computation of Triad Census

Configuration		Count
Intransitive Triads	Divergent	$\mathbf{1}'\left(\frac{1}{2}A'A - A^2 \circ A\right)\mathbf{1}$
	Convergent	$\mathbf{1}'\left(\frac{1}{2}AA' - A^2 \circ A\right)\mathbf{1}$
	Chains	$\mathbf{1}'\left(A^2 - A^2 \circ A' - A^2 \circ A\right)\mathbf{1}$
Transitive Triads	Cyclic	$\frac{1}{3}\mathbf{1}'\left(A^2 \circ A'\right)\mathbf{1}$
	Acyclic	$\mathbf{1}'\left(A^2 \circ A\right)\mathbf{1}$

intransitive triads, divergent triads are most likely, followed by chains and then convergent triads. Among transitive triads, acyclic triads are almost 20 times more likely than cyclic triads. This pattern is consistent across all years of data.

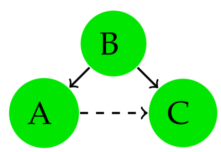
To assess dynamics of formation of relationships between pairs of firms that had a common connection in the previous period but were not directly connected, I count cases where pairs of firms that start trading in the current period were only indirectly connected in the previous period. Such occurrences are called triadic closures, cases where intransitive triads in the previous period turn into transitive triads in the current period. Closure of intransitive triads of the divergent and convergent types leads to formation of an acyclic transitive triad. Closure of chain intransitive triads can lead to either the formation of a cyclic or acyclic transitive triad. Figure 2.2 depicts various kinds of triadic closures. Let B denote a matrix such that the $(i,j)^{th}$ element takes the value 1 if i is a supplier to j in the current period but not in the previous period. Table 3 provides formulas for computing counts of different closures from network data. Table 4 reports the counts of such closures across all years of the data. It shows that divergent and convergent closures are more numerous than acyclic chain closures, followed by cyclic chain closures. This pattern is consistent across all years of data.

Table 2: Counts of Intransitive and Transitive Triads

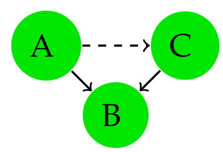
	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016
Intransitive:					
Divergent	1,541,767,883	1,674,068,239	1,847,644,020	2,285,340,927	2,391,444,882
Convergent	710,538,443	767,975,704	884,390,233	916,403,383	965,808,255
Chains	831,275,833	901,136,158	1,036,222,732	1,093,610,280	1,161,854,619
Transitive:					
Acyclic	47,210,129	48,624,599	53,535,783	55,098,166	59,459,796
Cyclic	2,727,048	2,770,515	3,192,921	3,025,801	3,238,061

Figure 2.2: Triadic Closures

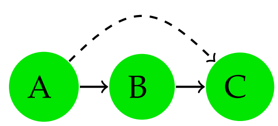
(a) Divergent Closure



(b) Convergent Closure



(c) Acyclic Chain Closure



(d) Cyclic Chain Closure

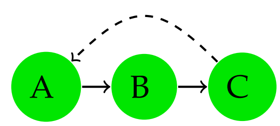


Table 3: Triadic Closure Census Formulas

Configuration	Count
Cyclic Chain Closure	$\mathbf{1}'\left(A^2 \circ B'\right)\mathbf{1}$
Acyclic Chain Closure	$\mathbf{1}'\left(A^2 \circ B\right)\mathbf{1}$
Divergent Closure	$\frac{1}{2}\mathbf{1}'\left(A'A \circ B\right)\mathbf{1}$
Convergent Closure	$\frac{1}{2}\mathbf{1}'\left(AA' \circ B\right)\mathbf{1}$

Table 4: Triadic Closure Counts

	2012-2013	2013-2014	2014-2015	2015-2016
Divergent Closure	13,091,156	13,712,393	14,058,856	15,532,098
Convergent Closure	12,281,334	12,520,390	13,202,401	
Cyclic Chain Closure	2,542,209	2,668,685	2,746,763	2,781,865
Acyclic Chain Closure	9,658,748	10,359,705	10,725,559	11,582,947

Table 5: Triad Census across Group Triads

Configuration	Count for (i, j, k)
Cyclic	$L_i' \left(A \times diag\left(L_j\right) \times A \circ A'\right) L_k$
Acyclic	$L_i' \left(A \times diag\left(L_j\right) \times A \circ A\right) L_k$
Chains	$L_i' A \times diag\left(L_j\right) \times A L_k$
Divergent	$\left(1 - \frac{1}{2}\mathbb{I}\left(i = k\right)\right) \left(L_i' A' \times diag\left(L_j\right) \times A L_k\right)$
Convergent	$\left(1 - \frac{1}{2}\mathbb{I}\left(i = k\right)\right) \left(L_i' A \times diag\left(L_j\right) \times A' L_k\right)$

Furthermore, these triads can be classified based on the location at which firms in the triad are located. Using information on the district where each firm is located, I count the number of triads (transitive and intransitive) in each triplet of districts. Let L be a matrix such that the $(i, j)^{th}$ element is 1 if i is located in j and 0 otherwise and L_r denote the r^{th}column of L. For any triplet of districts (i, j, k), the formulas for computing the number of triads are in Table 5.

2.3 Excess Prevalence of Transitive Triads

Having enumerated counts of triad configurations, I now turn to assess if they are more in number than expected. I proceed in two ways. First, I evaluate if such configurations occur in much larger numbers than would be predicted by a model where firm-to-firm connections are formed uniformly with the same probability. Second, I evaluate if firm pairs that have at least one common connection are more likely to be connected than those that do not.

Consider the Erdos-Renyi random graph as the baseline random network formation model. Much like the uniform distribution for random variables where all values in the support are drawn with equal probability, the Erdos-Renyi random graph is one where all links are formed with equal probability. Using the Erdos-Renyi random graph as a benchmark, I assess if the observed number of triads is more than what a Erdos-Renyi model would predict. Suppose all links between firms were formed with the same probability p. An estimate of this probability can be obtained as the ratio of the number of observed links and the number of possible links that could be formed between M firms. The latter is $\binom{M}{2}$ and the former is obtained from the data. With a group of three firms, there are three possible pairs of firms between whom links can be formed. This leads to four possible outcomes: $\{0, 1, 2, 3\}$ links. The probability of intransitive triads is $\binom{3}{2}p^2(1-p)$ and that of transitive triads is $\binom{3}{3}p^3$. With M firms, there are $\binom{M}{3}$ unique triplets of firms. The expected number of intransitive and transitive triads are therefore $\binom{3}{2}p^2(1-p)\binom{M}{3}$ and

Table 6: Expected Count of Triads in Erdos-Renyi Model

Configuration	Expected Count
Intransitive	$\binom{M}{3}\binom{3}{2}p^2(1-p)$
Transitive	$\binom{M}{3}\binom{3}{3}p^3$

Table 7: Expected Count of Triads in Erdos-Renyi Model with group heterogeneity

Configuration	Expected Count (i,j,k)
Intransitive	$\begin{cases} \left(\left(1-p_{ij}\right)p_{jk}p_{ik}+p_{ij}\left(1-p_{jk}\right)p_{ik}+p_{ij}p_{jk}\left(1-p_{ij}\right)\right)M_iM_jM_k & i\neq j\neq k \\ \left(2p_{ii}\left(1-p_{ii}\right)p_{ik}+p_{ii}^2\left(1-p_{ik}\right)\right)\binom{M_i}{2}M_j & i=j\neq k \\ 3p_{ii}^2\left(1-p_{ii}\right)\binom{M_i}{3} & i=j=k \end{cases}$
Transitive	$\begin{cases} p_{ij}p_{jk}p_{ik}M_iM_jM_k & i\neq j\neq k \\ p_{ii}^2p_{ik}\binom{M_i}{2}M_k & i=j\neq k \\ p_{ii}^3\binom{M_i}{3} & i=j=k \end{cases}$

$\binom{3}{3}p^3\binom{M}{3}$ respectively as reported in Table 6.

The top panel in Table 8 reports the overlikelihood of intransitive and transitive triads in the actual data relative to the Erdos-Renyi model. It shows that while intransitive triads are six times more likely to occur, transitive triads are 13,750 times more likely to occur. To account for heterogeneity across nodes, I consider a heterogeneous version of the Erdos-Renyi model where the probability of connecting varies by the pair of locations in which the firm pair is located or how connected the each firm is. I group firms into bins (based on location and connectivity) and allow for heterogeneous probability of link formation between a pair of groups. Table 7 reports the formula for counting the expected number of triads.

$$\widehat{p}_{ij} = \begin{cases} \frac{\text{\# connections}}{M_iM_j} & i\neq j \\ \frac{\text{\# connections}}{\binom{M_i}{2}} & i=j \end{cases}$$

In the first exercise, I allow the probability of connecting vary by the pair of locations that the firms are located in. The second panel in Table 8 reports the overlikelihood of triads when accounting for heterogeneity in probability of link formation by their location. It shows that while intransitive triads are 2300 times less likely in the model than observed in data, transitive triads are 600 times more likely. In the second exercise, I allow the probability of connecting vary by the pair of degree quantile that firms would fall into. The third panel in Table 8 reports the overlikelihood of triads when accounting for heterogeneity in probability of link formation by their degree quantile. It shows that while intransitive triads are 23 times less likely in the model than observed in data,

Table 8: Overlikelihood relative to Erdos-Renyi Random Graph

	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016
Baseline:					
Intransitive	6	6	6	7	6
Transitive	13078	13722	13723	14094	14141
Location Heterogeneity:					
Intransitive	$1/2207$	$1/2049$	$1/2375$	$1/2398$	$1/2545$
Transitive	630	623	629	571	586
Degree Heterogeneity:					
Intransitive	$1/22$	$1/22$	$1/24$	$1/23$	$1/24$
Transitive	78	79	89	94	98
Triadic Closure:					
Divergent Closure		390	399	399	
Convergent Closure		779	782	771	
Acyclic Chain Closure		1082	1137	1102	1131
Cyclic Chain Closure		285	293	282	272

transitive triads are 90 times more likely.

The bottom panel of Table 8 shows that divergent closures are 400 times more likely than the baseline Erdos-Renyi model. Convergent closures are 780 times more likely. Acyclic and cyclic chain closures are respectively 1100 and 300 times more likely.

I now turn to the next exercise, to evaluate if a pair of firms that have a common connection are more likely to directly trade with each other. Of the 23×10^6 first order connections between firms, 15×10^6 firms also simultaneously also have a second order connection between them. There are around 1.6×10^6 nodes in the network. This implies that there are 2.56×10^{12} unique pairs of firms. Of these 0.9×10^9 have at least one second order connection between them. These numbers imply the following. The unconditional probability of a link forming between a pair of firms is $\frac{23 \times 10^6}{2.56 \times 10^{12}} = 9 \times 10^{-6}$. Conditional on having a second order connection, the probability is $\frac{15 \times 10^6}{0.9 \times 10^9} = 16 \times 10^{-3}$. The conditional probability is therefore $\frac{16 \times 10^{-3}}{9 \times 10^{-6}} \approx 1800$ times the unconditional probability.

Furthermore, suppose we divide pairs of firms into two groups: one where the pairs have a second-order connection and one where they do not. The first group consists of 0.9×10^9. the probability of link formation between those firm pairs is $\frac{15 \times 10^6}{0.9 \times 10^9} = 16 \times 10^{-3}$. The second group consists of $2.56 \times 10^{12} - 0.9 \times 10^9 \approx 2.56 \times 10^{12}$ pairs, so the probability of a link in this group of firm pairs is $\frac{8 \times 10^6}{2.56 \times 10^{12}} = 3.125 \times 10^{-6}$. The link formation probability in the first group is therefore $\frac{16\times10^{-3}}{3.125\times10^{-6}} \approx 5120$ times the second group.

2.4 Network Proximity and Aggregate Trade Flows

Empirically, locations with higher geographic proximity trade more. Hence, in gravity models, locations with geographic proximity are typically modeled to trade more. Are there other factors, conditional on distance, that makes it more likely for location pairs to trade more? Is distance a proxy for other properties of the network? I explore a new notion of proximity between a pair of locations – through a third location. In particular, I count how many times it is the case that for three locations (o, r, d), a firm in o has contact in r which then has a contact in d. This can happen in four ways. First, the firm in o sells to a customer in r which itself sells to a customer in d – an outgoing chain from o to d through r. Second, the firm in o buys from a supplier in r which itself buys from a firm in d – an incoming chain that comes from d to o though r. Third, a firm in o and a firm in d could have a common seller in r – a divergent connection between o and d through r. Finally, a firm in o and a firm in d could have a common buyer in r – a convergent connection between o and d through r. Table 9 provides formulas counting such configurations across location triplets (o, r, d).

Table 10 reports the results of gravity regressions implied by the model in Eaton et al. (2013) but including variables measuring network proximity between locations. Network proximity between a pair of locations (o, d) is measured as the number of second-order connections (of each type) between o and d through all third locations r. Column (1) reports results of gravity regression with the usual distance and border variables that capture geographic proximity. Column (2) reports results when including network proximity variables. Column (3) reports results with modified network proximity variables that exclude second-order connections that either go through the origin or destination from the count. Focussing on the distance coefficient, the table shows that including network proximity variables almost halves the coefficient. Furthermore, among the various configurations, number of convergent and outgoing chain configurations are most positively predictive of trade. These results show that locations that are closer in the production network trade more.

Table 9: Second-order Connections between Locations

Configuration		Count for (o,r,d)
Incoming Chains	$o \leftarrow r \leftarrow d$	$L_o' A' D_r A' L_d$
Outgoing Chains	$o \rightarrow r \rightarrow d$	$L_o' A D_r A L_d$
Divergent	$o \leftarrow r \rightarrow d$	$\left(1 - \frac{1}{2}\mathbb{I}\{o = d\}\right) L_o' A' D_r A L_d$
Convergent	$o \rightarrow r \leftarrow d$	$\left(1 - \frac{1}{2}\mathbb{I}\{o = d\}\right) L_o' A D_r A' L_d$

3 Model

In this section, I describe a model of trade between multiple locations that accommodates heterogeneity in consumer preferences, heterogeneity in technological requirements of firms and arbitrary production networks. The model economy consists of many firms and households at many locations. Firms produce using local labor and intermediate inputs sourced from suppliers potentially spread across multiple locations. Trade between locations is subject to iceberg trade costs , that is, a firm producing at o needs to ship τ_{od} units of a good for one unit of good to arrive at d.

3.1 Technology and Market Structure

Firms' production processes involve combining labor and accomplishing a set of tasks by sourcing intermediate inputs from other firms. In particular, the production function for any firm b at location d is defined over labor and a discrete number of tasks (indexed by $k \in \mathcal{K}_d(b) \equiv \{1, \cdots, K_d(b)\}$) as:

$$y_d(b) = z_d(b) \left(\frac{l_d(b)}{1-\alpha_d}\right)^{1-\alpha_d} \left(\frac{\prod_{k \in \mathcal{K}_d(b)} m_d(b,k)^{1/K_d(b)}}{\alpha_d}\right)^{\alpha_d},$$

$$m_d(b,k) = \sum_{s \in \mathcal{S}_d(b)} m_{od}(s,b,k),$$

where $l_d(b)$ is the amount of labor input used by firm b, $m_d(b,k)$ is the quantity of materials utilized to accomplish task k, $z_d(b)$ is the idiosyncratic Hicks-neutral productivity with which firm b produces, and $K_d(b)$ is the number of tasks in the production function of firm b.

Among all the firms in the economy, firm b encounters only a few and can source intermediate inputs to accomplish tasks only from those firms. In particular, it encounters a potential supplier s with probability $\frac{\lambda \phi_{od}(s)}{M}$ via independent Bernoulli trials. The re-

Table 10: Gravity Regressions

Dependent Variable: Aggregate Trade Share between (o, d)					
	(1)	(2)	(3) $r \notin \{o, d\}$	(4)	(5) $r \notin \{o, d\}$
log(distance)	-0.712	0.030	-0.325	-0.708	-0.332
	(0.111)	(0.037)	(0.050)	(0.045)	(0.049)
interstate	-2.125	-0.256	-0.884	-2.128	-0.853
	(0.197)	(0.098)	(0.133)	(0.090)	(0.132)
interdistrict	-1.852	-0.401	-1.124	-1.839	-1.149
	(0.222)	(0.075)	(0.083)	(0.082)	(0.083)
neighbor	0.251	0.000	0.318	0.248	0.326
	(0.169)	(0.040)	(0.049)	(0.052)	(0.049)
Sectoral Correlation				0.062	-0.187
				(0.063)	(0.056)
# Convergent		0.388	0.476		0.482
		(0.032)	(0.037)		(0.038)
# Divergent		0.048	0.044		0.057
		(0.038)	(0.041)		(0.041)
# Outgoing Chains		0.820	0.424		0.420
		(0.049)	(0.049)		(0.049)
# Incoming Chains		-0.173	-0.167		-0.172
		(0.029)	(0.036)		(0.037)
Fixed Effects:					
Origin×Year	✓	✓	✓	✓	✓
Destination×Year	✓	✓	✓	✓	✓
Pseudo R^2	0.073	0.097	0.083	0.073	0.082
Squared Correlation	0.793	0.858	0.814	0.792	0.815
# observations	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$

Note. Standard errors in parentheses, two-way clustered by origin–year and destination–year. Observations pertain to all bilateral pairs between 141 districts for 5 years. The distance between district pairs is calculated as the distance between their centroids. A district's distance to itself is calculated as the radius of the circle with the same area as the district. Estimation is carried out using a multinomial PML specification a la Eaton et al. (2013). Two-way clustering is done as in Cameron et al. (2011). Pseudo R^2 is calculated as in McFadden (1974).

stricted set of potential suppliers, denoted by $\mathcal{S}_d(b)$, is therefore completely determined as the outcome of these Bernoulli trials for meeting each firm and is common for all tasks. While outputs of potential suppliers are perfectly substitutable for accomplishing any task, they differ in their suitability for the task in question, captured by their respective match-specific productivities. For each of its tasks, firm b selects the supplier that offers the lowest effective price. For simplicity, I assume that firms engage in marginal cost pricing behavior when sourcing inputs.

3.2 Cost Minimization and Input Sourcing

I now turn to firms' cost minimization problem. Selecting the cost-minimizing input bundle consists of choosing not only who to source inputs from but also how much to buy from each of them. For any task k in firm b's production function, the cost-effectiveness of a supplier s from location o in $\mathcal{S}_d(b)$ depends on three factors: (a) the marginal cost of s, denoted $c_o(s)$; (b) the trade cost faced by s of shipping goods to d, τ_{od}; and (c) the match-specific productivity when b utilizes the output of s to accomplish the task, denoted by $a_{od}(s, b, k)$. In particular, firm b chooses the supplier that offers the cheapest price, that is,

$$s_d^*(b,k) = \arg \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \right\}. \tag{3.1}$$

Now, taking wage w_d and effective prices $\{p_d(b,k) : k \in \mathcal{K}_d(b)\}$ (defined below) as given, the firm's unit cost function is given by:

$$c_d(b) = \frac{w_d^{1-\alpha_d} \left(\prod_{k \in \mathcal{K}_d(b)} p_d(b,k)^{1/K_d(b)} \right)^{\alpha_d}}{z_d(b)}, \tag{3.2}$$

where $p_d(b,k)$ is determined according the following equation:

$$p_d(b,k) = \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \right\}. \tag{3.3}$$

3.3 Closing the Model

Household Preferences

Households are modeled analogously with tasks in their utility function. They encounter potential suppliers and select the most cost-effective suppliers for each task similar to firms sourcing inputs. Each household supplies one unit of labor inelastically to local firms and receives labor income. Firms rebate any profits to local households.

Equilibrium Definition

Let $\sigma \equiv \{\boldsymbol{z}, \boldsymbol{\tau}, \boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{K}}, \boldsymbol{a}\}$ denote the aggregate state of the economy. Here $\boldsymbol{z}$ denotes the vector of idiosyncratic productivities of firms, $\boldsymbol{\tau}$ denotes the vector of trade costs across all pairs of locations, $\boldsymbol{\mathcal{S}}$ denotes the sets of potential suppliers of all firms and households, $\boldsymbol{\mathcal{K}}$ denotes the sets of tasks of all firms and households, and $\boldsymbol{a}$ denotes the vector of all match-specific productivities. All of these objects are exogenous. An equilibrium in this economy is an allocation and a price system such that (a) households and firms select suppliers for tasks; (b) firms set prices for other firms and households under marginal cost pricing; (c) households maximize utility; (d) firms minimize costs; and (e) market clears for each firm's goods and for labor at each location. This completes description of the economic environment in the model.[2]

4 Estimation and Results

4.1 Taking Model to Data

To map the model to micro-data on firm-to-firm sales for estimation, I proceed in four steps. First, I utilize the recursive representation of network formation between firms to cast it as a quasi- dynamic programming problem. Second, I show that the model delivers closed-form characterization of conditional choice probabilities in this quasi-dynamic discrete choice setting. Third, I describe how these conditional choice probabilities coupled with multiple discrete choice across tasks lead to a multinomial logit model of supplier choice. Finally, I tackle the computational burden imposed by the high-dimensionality of the non-linear estimation problem by exploiting special features of the multinomial likelihood specification. The resulting estimation framework is scalable and circumvents computational difficulties pervasive in estimation of network formation models with large numbers of firms.

4.1.1 Conditional Choice Probabilities & Firm-to-Firm Trade

I begin by casting network formation between firms as a quasi-dynamic programming problem. In particular, combining equations (3.2) and (3.3), I find that marginal cost of

[2] A detailed description is provided in Appendix A

any firm b admits the following recursive representation.

$$c_d(b) = \frac{w_d^{1-\alpha_d}}{z_d(b)} \times \prod_{k \in \mathcal{K}_d(b)} \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{\tau_{od}}{a_{od}(s,b,k)} \times c_o(s) \right\}^{\frac{\alpha_d}{K_d(b)}} \tag{4.1}$$

This representation is akin to a setting with dynamic discrete choice (albeit with multiple discrete choice). The estimands in this estimation problem are trade costs $\{\tau_{od} : (o,d) \in \mathcal{J}^2\}$ which are exogenous and firms' marginal costs $\{c_o(s) : s \in \mathcal{M}\}$ which are endogenously determined, unobserved in the data and run into millions. I utilize the conditional choice probability approach to estimate the model following Hotz and Miller (1993). In this context, conditional choice probabilities are the probabilities with which any given supplier s is chosen for any one of the buyer b's tasks conditional on its marginal cost being $c_o(s)$. I proceed to show next that the model delivers closed-form predictions for these probabilities.

I turn to expressions for conditional choice probabilities and hence predictions for firm-to-firm trade. I assume that match-specific productivities are drawn independently for all potential suppliers for each of the tasks in firms' production functions from a Pareto distribution as stated in the following assumption.

Assumption 1. *Match-specific productivities are drawn independently according to the following Pareto distribution:*

$$F_a(a) = 1 - \left({}^{a}/_{a_0}\right)^{-\zeta}.$$

In a sufficiently large economy such that $0 < {}^{\lambda}/_{M} \ll 1$, $|\lambda a_0^{\zeta} - 1| < \varepsilon_1$, and $|a_0| < \varepsilon_2$ for arbitrarily small values of ε_1 and ε_2 one can obtain closed-form expressions for conditional choice probabilities. Recall from equation (3.1) that firms choose suppliers for tasks based on suppliers' marginal costs, trade costs faced by them, and match-specific productivities associated with the task under consideration. While trade costs $\boldsymbol{\tau}$ constitute σ_0, match-specific productivities are unknown and suppliers' marginal costs $c_o(s)$ are determined endogenously. I therefore characterize conditional choice probabilities for supplier choice, i.e., probabilities for choice of supplier conditional on its marginal cost but in expectation over match-specific productivities that are yet to be realized. Let $\pi_{od}^0(s,b)$ denote the probability with which firm b selects firm s for any one of its tasks. Prior to encountering and realizing match-specific productivities for each task, the probability of firm s getting selected for any one of the tasks by firm b is common across all tasks. That is, $\pi_{od}^0(s,b) = \pi_{od}^0(s,b,k) = \mathbb{E}_{\{\sigma_1\}}\left[\mathbf{1}\left\{s = s_d^*(b,k) \mid \sigma_0, \sigma_1\right\}\right]$ where the expectation operator is over all realizations of σ_1. The following proposition provides expressions for conditional choice probabilities $\pi_{od}^0(s,b)$.

Proposition 1. *For any realization of σ_0, conditional on firm s's marginal cost being $c_o(s)$, the probability with which any firm b located in d selects firm s located in o for any given task is*

$$\pi^0_{od}(s,b) = \frac{c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta}}. \tag{4.2}$$

Proof. See Appendix B.1. □

The tractable expressions for firm-to-firm trade in Proposition 1 give rise to transparent estimating equations for the model, to which I turn next.

4.1.2 A Multinomial Logit Model of Supplier Choice

I reformulate the economic model developed so far as a multinomial logit model of supplier choice for tasks of each of the firms and estimate it semi-parametrically with seller fixed effects, seller-destination fixed effects and origin-destination fixed effects. Origin-destination fixed effects correspond to a structural gravity specification for estimating trade frictions. Trade frictions are then estimated by projecting bilateral fixed effects on observables that capture geographic proximity such as distance and borders etc as well as on observables that capture network proximity.

Making use of Proposition 1, the estimating equation can be expressed as a multinomial logit function:

$$\mathbb{E}\left[\pi_{od}(s,b)\right] = \frac{c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta}} \tag{4.3}$$

Formally, the estimation problem is as follows:

$$\Delta^* = \arg\max_{\Delta} \frac{1}{M}\sum_{b\in\mathcal{M}} \ln f_{\text{MNL}}\left(\mathbb{D} \mid \Delta\right), \tag{4.4}$$

$$f_{\text{MNL}}\left(\mathbb{D} \mid \Delta\right) \propto \prod_{s\in\mathcal{M}} \left(\frac{c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta}}\right)^{\pi_{od}(s,b)},$$

where

$$\Delta \equiv \left\{\left\{c_o(s)^{-\zeta} : s\in\mathcal{M}\right\}, \left\{\phi_{od}(s) : (s,d)\in\mathcal{M}\times\mathcal{J}\right\}, \left\{\tau_{od}^{-\zeta} : (o,d)\in\mathcal{J}^2\right\}\right\} \text{ and}$$
$$\mathbb{D} \equiv \left\{\pi_{od}(s,b) : (s,b)\in\mathcal{M}^2\right\}.$$

The above specification with fixed effects however presents a problem of perfect multicollinearity in regressors. Note that dummy variables associated with $\left\{c_o(s)^{-\zeta} : s \in \mathcal{M}_o\right\}$ and $\left\{\tau_{od}^{-\zeta} : d \in \mathcal{J}\right\}$ are collinear for all such locations o. Hence, I make the following normalizations so that these fixed effects are identified up to scale. For all $s \in \mathcal{M}_o, o \in \mathcal{J}$, let $c_o(s) = c_o\widetilde{c}_o(s)$ and $\phi_{od}(s) = \overline{\phi_{od}}\widetilde{\phi}_{od}(s)$ be such that

$$\left(\sum_{s\in\mathcal{M}_o} \widetilde{c}_o(s)^{-\zeta}\widetilde{\phi}_{od}(s)\right)^{-1/\zeta} = 1,$$

$$\sum_{d\in\mathcal{J}} \frac{\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} M_d}{\sum_{d\in\mathcal{J}} \frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} M_d} \widetilde{\phi}_{od}(s) = 1.$$

The multinomial logit specification is problematic because of two reasons. On one hand, firms' marginal costs are endogenously determined and unobserved. They are estimated semiparametrically as firm fixed effects. Since there are a large number of firms in the economy, estimation would typically require high-dimensional non-linear optimization over a very large number of parameters to solve for the estimates. This can be computationally infeasible using standard Newton methods when the number of fixed effects runs into millions. On the other hand, estimation of a generalized linear model with millions of fixed effects leads to incidental parameters bias in the lower-dimensional estimands.

However, these issues are taken care of by appealing to several special features of the multinomial likelihood function. First, estimates can be obtained using the Poisson likelihood function with additional fixed effects (see Baker (1994); Taddy (2015)). Second, Poisson likelihood estimation automatically satisfies adding up constraints implied by the model (see Fally (2015)). Third, Poisson likelihood specification allows solving for fixed effects in closed-form (for example, see Hausman et al. (1984)). Finally, subsequent estimation of trade frictions using bilateral fixed effects does not suffer from the incidental parameters problem and hence can be conducted through the conditional maximum likelihood approach.

Fixed Effects and Structural Gravity The first order conditions implied by the likelihood maximization problem in equation (4.4) can be solved to obtain closed-form estimators for fixed effects as described in the proposition below.

Proposition 2. *The estimates from equation (4.4) are given by:*

$$\left(\widetilde{c}_o(s)^{-\zeta}\right)^* = \frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{\sum_{b\in\mathcal{M}} \pi_{od}(\bullet,b)} \quad \forall s \in \mathcal{M}, \tag{4.5}$$

$$\left(\widetilde{\phi}_{od}(s)^{-\zeta}\right)^* = \frac{\frac{\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{\sum_{b\in\mathcal{M}_d} \pi_{od}(\bullet,b)}}{\frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{\sum_{b\in\mathcal{M}} \pi_{od}(\bullet,b)}} \quad \forall (s,d) \in \mathcal{M}\times\mathcal{J}, \tag{4.6}$$

$$\left(\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}}\right)^* = \frac{1}{M_d}\sum_{b\in\mathcal{M}_d} \pi_{od}\left(\bullet,b\right) \quad \forall (o,d) \in \mathcal{J}^2 \tag{4.7}$$

where $\pi_{od}\left(\bullet,b\right) \equiv \sum_{s\in\mathcal{M}_o} \pi_{od}\left(s,b\right)$.

Proof. See Appendix B.2. □

Trade Frictions and Conditional Choice Probabilities With seller and seller-destination fixed effects out of the way, thanks to equations (4.5), trade frictions can now be estimated by projecting bilateral origin-destination fixed effects (from equation (4.7)) on bilateral observables such as distance, borders etc., similar to gravity regressions, in additional to variables that capture network proximity with the following estimating equation:

$$\mathbb{E}\left[\left(\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}}\right)^*\right] = \frac{\exp\left(\ln\left(c_o^{-\zeta}\right) + \boldsymbol{\omega}_{od}'\gamma + \boldsymbol{X}_{od}'\boldsymbol{\beta}\right)}{\sum_{o'} \exp\left(\ln\left(c_{o'}^{-\zeta}\right) + \boldsymbol{\omega}_{od}'\gamma + \boldsymbol{X}_{o'd}'\boldsymbol{\beta}\right)}. \tag{4.8}$$

This delivers estimates of origin fixed effects $\left(c_o^{-\zeta}\right)^*$ and trade frictions arising from geography $\left(\tau_{od}^{-\zeta}\right)^* = \exp\left(\boldsymbol{X}_{od}'\boldsymbol{\beta}^*\right)$ and network structure $\left(\overline{\phi_{od}}\right)^* = \exp\left(\boldsymbol{\omega}_{od}'\gamma^*\right)$. Estimates of conditional choice probabilities are then obtained from seller and seller-destination fixed effects and fitted shares from the gravity regressions. Formally, the estimates of conditional choice probabilities are given by

$$\pi_{od}^*(s,b) = \left(\widetilde{c}_o(s)^{-\zeta}\right)^* \cdot \left(\widetilde{\phi}_{od}(s)\right)^* \cdot \pi_{od}^*(\bullet,b), \tag{4.9}$$

$$\pi_{od}^*(\bullet,b) = \frac{\left(c_o^{-\zeta}\right)^*\left(\overline{\phi_{od}}\right)^*\left(\tau_{od}^{-\zeta}\right)^*}{\sum_{o'\in\mathcal{J}}\left(c_{o'}^{-\zeta}\right)^*\left(\overline{\phi_{o'd}}\right)^*\left(\tau_{o'd}^{-\zeta}\right)^*}. \tag{4.10}$$

4.2 Estimation Results

Table 11 reports results of gravity regressions from equation (4.10). Columns (1) and (4) show that including network proximity variable leads to a 40% decrease in the distance coefficient. Among network configurations, outgoing chains and convergent configurations increase most the likelihood of trade between locations. Incoming chains negatively affect trade while the effect of divergent configurations is insignificant. Column (5) includes sectoral correlation in economic activity between origin and destination to capture economic similarity. Despite its inclusion, the results from column (4) persist.

Using the estimated trade frictions from Table 11, Column (3), I conduct a Shapley decomposition (see Shorrocks (2013)) of trade frictions into two components: geographic proximity and network proximity. Table 12 shows that network proximity explains a dominant majority of trade frictions.

5 Aggregation

For aggregation and counterfactual analysis, I adopt the large economy model due to Al-Najjar (2004) which is characterized by a sequence of finite but increasingly large economies $\{\mathcal{E}_t : t \in \mathbb{N}\}$ that progressively discretizes the unit continuum. Along the sequence as the economy becomes more discretized, I make additional assumptions so that the model has a well-defined limit. The probability of meeting potential suppliers increases, i.e., $\lim_{t\to\infty} \lambda_t = \infty$, but at a rate slower than that at which the economy is discretized, i.e., $\lim_{t\to\infty} \frac{\lambda_t}{M_t} = 0$. At the same time, match-specific productivities are drawn from stochastically worse distributions as $\lim_{t\to\infty} a_{0,t} = 0$. While the number of potential suppliers grows arbitrarily large and the match-specific productivity associated with any single supplier is drawn from a stochastically worse distribution, the limit is well behaved because the probability of encountering a supplier with match-specific productivity greater than value a does not change in the limiting economy, i.e., $\lim_{t\to\infty} \lambda_t a_{0,t}^{\zeta} = 1$. Furthermore, the economy becomes discretized in a manner such that the proportion of firms and households at every location is non-zero and finite.

I now proceed to characterize effective prices $\boldsymbol{p}(\sigma)$ and wages $\boldsymbol{w}(\sigma)$ in equilibrium in the limiting economy, i.e., $\lim_{t\to\infty} \mathcal{E}_t$.

Table 11: Model-Consistent Gravity Regressions

Dependent Variable: Average Trade Share between (o, d)					
	(1)	(2)	(3) $r \notin \{o, d\}$	(4)	(5) $r \notin \{o, d\}$
log(distance)	-0.99	-0.116	-0.639	-0.942	-0.609
	(0.045)	(0.031)	(0.030)	(0.045)	(0.044)
interstate	-2.579	-0.673	-1.524	-2.351	-1.242
	(0.088)	(0.062)	(0.081)	(0.088)	(0.091)
interdistrict	-2.262	-0.783	-1.502	-2.207	-1.504
	(0.067)	(0.047)	(0.053)	(0.070)	(0.066)
neighbor	0.517	0.256	0.519	0.505	0.523
	(0.047)	(0.027)	(0.033)	(0.048)	(0.042)
Sectoral Correlation				0.226	-0.024
				(0.060)	(0.046)
# Convergent		0.385	0.519		0.494
		(0.029)	(0.026)		(0.035)
# Divergent		0.071	0.029		0.049
		(0.033)	(0.029)		(0.032)
# Outgoing Chains		0.799	0.302		0.302
		(0.046)	(0.032)		(0.036)
# Incoming Chains		-0.138	-0.217		-0.203
		(0.029)	(0.030)		(0.034
Fixed Effects:					
Origin×Year	✓	✓	✓	✓	✓
Destination×Year	✓	✓	✓	✓	✓
Pseudo R^2	0.119	0.139	0.125	0.119	0.125
Squared Correlation	0.887	0.964	0.908	0.887	0.908
# observations	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$	$141^2 \times 5$

Note. Standard errors in parentheses, two-way clustered by origin–year and destination–year. Observations pertain to all bilateral pairs between 141 districts for 5 years. The distance between district pairs is calculated as the distance between their centroids. A district's distance to itself is calculated as the radius of the circle with the same area as the district. Estimation is carried out using a multinomial PML specification from equation (4.8). Two-way clustering is done as in Cameron et al. (2011). Pseudo R^2 is calculated as in McFadden (1974).

Table 12: Components of Trade Frictions

	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016	Overall
Network	75.13%	75.20%	73.61%	73.51%	73.65%	74.32%
Geographic	24.87%	24.80%	26.39%	26.49%	26.35%	25.68%

5.1 Market Access & Distributions of Effective Prices

With marginal cost pricing, the distribution of effective prices faced by a firm for any of its tasks is characterized by the distribution of the offer with the lowest effective cost to the supplier. The following proposition provides the distribution of effective prices in the limiting economy.

Proposition 3. *For any realization of σ_0, the effective prices of materials used by firm b to accomplish any task, $p_d(b,k)$ converge to the following distribution as $t \to \infty$:*

$$F_{p_d}(p) = 1 - e^{-A_d p^{\zeta}},$$

where $\boldsymbol{A} \equiv \{A_d : d \in \mathcal{J}\}$ is the unique positive solution to the following fixed point problem:

$$A_d = \sum_{o \in \mathcal{J}} \overline{\phi_{od}} \tau_{od}^{-\zeta} \overline{z_o^{\zeta}} \mu_o w_o^{-\zeta(1-\alpha_o)} \mathbb{E}\left[\Gamma\left(1 - \frac{\alpha_o}{K_o(\cdot)}\right)^{K_o(\cdot)}\right] A_o^{\alpha_o}, \tag{5.1}$$

where μ_o denotes the proportion of firms at o and $\overline{z_o^{\zeta}} = \mathbb{E}\left[z_o(s)^{\zeta}\right]$.[3]

Proof. See Appendix C.2. □

While the effective price faced by individual firms varies across realizations of σ_1, the cross-sectional distribution in the limit economy does not. These distributions are parametrized by a scale parameter A_d and a shape parameter ζ. Market access, given by A_d, is a key object of interest because it summarizes the probabilistic access of firms at d to inputs from all locations. The functional form suggests that firms at a location with higher market access face stochastically lower effective prices. Specifically, if $A_d > A_{d'}$, the distribution $F_{p_{d'}}(\cdot)$ first-order stochastically dominates $F_{p_d}(\cdot)$.

5.2 Relative Wages in Trade Equilibrium

To define relative wages in trade equilibrium, I begin by characterizing sourcing probabilities, that is, the probability with which any buyer sources inputs from location o for any one its tasks. Conditional choice probabilities of supplier choice naturally aggregate to sourcing probabilities, that is, sourcing probabilities can be obtained as the sum of conditional choice probabilities associated with all the suppliers located at o. Conditional

[3] The gamma function $\Gamma(\cdot)$ is defined as $\Gamma(x) = \int_0^{\infty} e^{-x} m^{x-1} dm$.

choice probabilities from Proposition 1 together with properties of the cross-sectional distributions of effective prices from Proposition 3 lead to the next proposition. This proposition characterizes sourcing probabilities across origins by firm b, denoted by $\pi^0_{od}(\bullet, b)$.

Proposition 4. *For any realization of σ_0, the probability with which any firm b located in d selects a supplier from o for any given task is*

$$\pi^0_{od}(\bullet, b) = \frac{\mu_o w_o^{-\zeta(1-\alpha_o)} \overline{z^\zeta_o} \mathbb{E}\left[\Gamma\left(1 - \frac{\alpha_o}{K_o(\cdot)}\right)^{K_o(\cdot)}\right] A_o^{\alpha_o} \overline{\phi_{od}} \tau_{od}^{-\zeta}}{A_d}. \tag{5.2}$$

Proof. See Appendix C.3 □

These sourcing probabilities are independent of the identity of the buyer at the destination and therefore can be written as $\pi^0_{od}(\bullet, -)$. In the limiting economy, the average sourcing share across all buyers in the limiting economy coincides with the expected value given by equation (5.2). This however does not mean that the sourcing shares across individual buyers are identical either in the finite economy or the limiting economy. Buyers at a destination may very well differ in their sourcing shares whether in the finite economy or the limiting economy. Formally, the law of large numbers implies that in the limiting economy,

$$\frac{1}{M_d} \sum_{b \in \mathcal{M}_d} \pi_{od}(\bullet, b) \xrightarrow{t \to \infty} \pi^0_{od}(\bullet, -). \tag{5.3}$$

I now turn to characterizing relative wages in the trade equilibrium in the limiting economy. The following proposition shows that relative wages in the limiting economy can be obtained as a solution to the system of equations (5.4).

Proposition 5. *For any realization of $\sigma \equiv \{\sigma_0, \sigma_1\}$, $\boldsymbol{w} \equiv \{w_d : d \in \mathcal{J}\}$ solves the following system of equations:*

$$\frac{w_o L_o}{1 - \alpha_o} = \sum_{d \in \mathcal{J}} \pi^0_{od}(\bullet, -) \frac{w_d L_d}{1 - \alpha_d}. \tag{5.4}$$

Further, for any σ and σ' such that $\sigma_0 = \sigma'_0$ and $\sigma_1 \neq \sigma'_1$:

$$\boldsymbol{w} = \boldsymbol{w}'. \tag{5.5}$$

Proof. See Appendix C.4. □

The above proposition also shows that, for any given realization of σ_0, relative wages are invariant across all networks realized for all values of σ_1. This concludes the characterization of equilibrium wages and brings us to the definition of the trade equilibrium below.

Definition 1. For any given σ_0, the trade equilibrium in the limiting economy is defined as the vector of wages $\boldsymbol{w}$ such that (a) market access at each location satisfies equation (5.1); (b) trade shares coincide with sourcing probabilities in equation (5.2) and (c) the market clearing condition in equation (5.4) holds.

6 Quantitative Analysis (in progress)

6.1 Computation of Counterfactual Outcomes

I operationalize Propositions 3, 4, and 5 for counterfactual analysis by expressing them in changes. The following definition states that and motivates the algorithm for evaluating counterfactual outcomes in response to shocks that derive from a change in the aggregate state σ_0 to σ_0'.

Definition 2. For any change in aggregate state σ_0 to σ_0', equilibrium change in wages $\widehat{\boldsymbol{w}} \equiv \{\widehat{w}_d : d \in \mathcal{J}\}$ and welfare $\widehat{\boldsymbol{V}} \equiv \left\{\widehat{V}_d : d \in \mathcal{J}\right\}$ are characterized the following system of equations for all realizations of σ_1 or σ_1':[4]

$$
\begin{aligned}
\widehat{A}_d &= \sum_o \pi_{od} \widehat{\tau_{od}}^{-\zeta} \widehat{\phi_{od}} \widehat{w}_o^{-\zeta(1-\alpha_o)} \widehat{A}_o^{\alpha_o} \\
\widehat{\phi_{od}} \phi_{od} &= g\left(\left\{\pi_{od}, \widehat{\tau_{od}}^{-\zeta}, \widehat{\phi_{od}}\right\}_{o,d}, \left\{\widehat{w}_o, \widehat{A}_o, \alpha_o, \left\{\widetilde{c}_o(s)^{-\zeta}\right\}_{s\in o}\right\}_o\right) \\
\widehat{\pi^0_{od}} &= \frac{\widehat{\tau_{od}}^{-\zeta} \widehat{\phi_{od}} \widehat{w}_o^{-\zeta(1-\alpha_o)} \widehat{A}_o^{\alpha_o}}{\widehat{A}_d} \\
\frac{\widehat{w}_o w_o L_o}{1-\alpha_o} &= \sum_d \widehat{\pi^0_{od}(\bullet, -)} \pi^0_{od}(\bullet, -) \frac{\widehat{w}_d w_d L_d}{1-\alpha_d} \\
\widehat{V}_d &= \widehat{w}_d \widehat{A}_d^{1/\zeta}
\end{aligned}
$$

where $\widehat{\delta} \equiv \left\{\widehat{\delta}_{od} : (o,d) \in \mathcal{J}^2\right\}$ is function of shocks that capture the resultant effect of change from σ_0 to σ_0'.

[4]The expression for welfare changes is derived in Appendix D.1.

With this definition of the equilibrium in changes in the limiting economy, aggregate and firm-level counterfactual outcomes in the limiting economy are computed in three steps. First, I evaluate aggregate and firm-level outcomes such as intensity of use and sales in the limiting economy in the initial state. Second, I evaluate changes in aggregate outcomes when going from the initial state to the counterfactual state. This is done using a tâtonnement algorithm similar to Alvarez and Lucas (2007) and Dekle et al. (2008). Finally, I evaluate aggregate and firm-level outcomes in the limiting economy in the counterfactual state. Details of the procedure are as follows:

Suppose $\phi_{od} \equiv \exp\left(\sum_{m\in\{I,O,D,C\}} \gamma_m \left(\omega_{od,m}\right)\right)$where

$$\begin{aligned} \omega_{od,I} &= \#\text{ incoming chains}_{od} \\ \omega_{od,O} &= \#\text{ outgoing chains}_{od} \\ \omega_{od,D} &= \#\text{ divergent}_{od} \\ \omega_{od,C} &= \#\text{ convergent}_{od} \end{aligned}$$

and $\{\gamma_m : m \in \{I,O,D,C\}\}$ are the correspoding coefficients. For any change in σ_0, $\widehat{\boldsymbol{\delta}} \equiv \left\{\widehat{\delta}_{od} : (o,d) \in \mathcal{J} \times \mathcal{J}\right\}$, one can solve for change in wages $\widehat{\boldsymbol{w}} \equiv \{\widehat{w}_d : d \in \mathcal{J}\}$ with the following tâtonnement algorithm for some positive constant μ and tolerance value tol:

1. Start with a guess for the vector of change in wages, $\widehat{\boldsymbol{w}}^{(0)}$ and
2. For the vector of wage changes, in the t^{th} iteration $\widehat{\boldsymbol{w}}^{(t)}$, compute change in market access and endogenous trade costs as the solution to the following system of equations:

$$\begin{aligned} \widehat{A}_d^{(t)} &= \sum_o \pi_{od}\widehat{\tau}_{od}^{-\zeta}\widehat{\phi}_{od}\left(\widehat{w}_o^{(t)}\right)^{-\zeta(1-\alpha_o)}\left(\widehat{A}_o^{(t)}\right)^{\alpha_o} \\ \widehat{\phi}_{od}^{(t)} &= \frac{\exp\left(\sum_{m\in\{I,O,D,C\}} \gamma_m \left(\omega_{od,m}^{(t)}\right)\right)'}{\phi_{od}} \\ \left(\omega_{ord,I}^{(t)}\right)' &= \mathbb{E}\left[\left(\rho_{ro}^{(t)}(\cdot)\right)'\right]\mathbb{E}\left[\left(\rho_{dr}^{(t)}(\cdot)\right)'\right] \\ \left(\omega_{od,O}^{(t)}\right)' &= \mathbb{E}\left[\left(\rho_{or}^{(t)}(\cdot)\right)'\right]\mathbb{E}\left[\left(\rho_{rd}^{(t)}(\cdot)\right)'\right] \\ \left(\omega_{od,D}^{(t)}\right)' &= \mathbb{E}\left[\left(\rho_{ro}^{(t)}(\cdot)\right)'\left(\rho_{rd}^{(t)}(\cdot)\right)'\right] \\ \left(\omega_{od,C}^{(t)}\right)' &= \mathbb{E}\left[\left(\rho_{or}^{(t)}(\cdot)\right)'\right]\mathbb{E}\left[\left(\rho_{dr}^{(t)}(\cdot)\right)'\right] \end{aligned}$$

$$\left(\rho_{od}^{(t)}(s)\right)' = \frac{1 - \exp\left(-\kappa_d \left(\widetilde{c}_o(s)^{-\zeta}\right) \pi_{od} \widehat{\tau}_{od}^{-\zeta} \widehat{\phi}_{od} \left(\widehat{w}_o^{(t)}\right)^{-\zeta(1-\alpha_o)} \left(\widehat{A}_o^{(t)}\right)^{\alpha_o} / \widehat{A}_d^{(t)}\right)}{1 - \exp\left(-\kappa_d\right)}$$

3. Compute counterfactual sourcing probabilities as:

$$\left(\pi_{od}^{(t)}\right)' = \pi_{od}^{(t)} \frac{\widehat{\tau}_{od}^{-\zeta} \widehat{\phi}_{od} \left(\widehat{w}_o^{(t)}\right)^{-\zeta(1-\alpha_o)} \left(\widehat{A}_o^{(t)}\right)^{\alpha_o}}{\widehat{A}_d^{(t)}}$$

4. Compute excess demand for labor $\boldsymbol{Z}\left(\widehat{\boldsymbol{w}}^{(t)}\right) \equiv \left\{Z_o\left(\widehat{\boldsymbol{w}}^{(t)}\right) : o \in \mathcal{J}\right\}$ as:

$$Z_o\left(\widehat{\boldsymbol{w}}^{(t)}\right) = \frac{1-\alpha_o}{w_o L_o} \sum_d \left(\pi_{od}^{(t)}\right)' \widehat{w}_d^{(t)} \frac{w_d L_d}{1-\alpha_d} - \widehat{w}_o$$

5. Update the vector of change in wages as $\widehat{\boldsymbol{w}}^{(t+1)} \leftarrow \widehat{\boldsymbol{w}}^{(t)} + \mu \boldsymbol{Z}\left(\widehat{\boldsymbol{w}}^{(t)}\right)$.

6. If $\|\widehat{\boldsymbol{w}}^{(t+1)} - \widehat{\boldsymbol{w}}^{(t)}\| > tol$, go back to (2), else end.

Welfare changes can then be computed as $\widehat{V}_d = \widehat{w}_d^{(\infty)} \left(\widehat{A}_d^{(\infty)}\right)^{\frac{1}{\zeta}}$.

The counterfactual outcomes thus computed for the limiting economy correspond to the expected value of outcomes for the finite economy in the counterfactual state since the limiting economy is a continuum approximation of the finite economy. In contrast to standard trade models, trade frictions are determined endogenously in the model here. This is because firm-to-firm connections are first formed due to geographic proximity which then leads to formation of further connections due to network proximity and so on. The above procedure accounts for endogenous determination of trade frictions. In the exact hat algebra approach (Dekle et al. (2008)) one solves for the fixed point in terms of changes in wages and market access in response to shocks through a tatonnement algorithm. The difference here is that the model leads to an additional fixed point equation for trade frictions which needs to be solved simultaneously. Owing to the non-linear structure of equations that solve for changes in market access, wages and trade frictions simultaneously, the impact of trade costs shocks on trade shares is non-linear unlike the class of models studied in Arkolakis et al. (2012).

7 Conclusion

This paper proposes a new origin of trade frictions: proximity on firm-to-firm production networks. Using rich administrative data on firm-to-firm linkages from India, I document that triads are excessively prevalent in firm-to-firm production networks. Furthermore, a firm-to-firm linkage is more likely to be part of a transitive triad than not. I find that proximity through the production network is an important determinant of trade frictions. I develop a quantitative general equilibrium model of network formation between spatially distant firms. The model aggregates to structural gravity and features endogenous trade frictions unlike standard trade models. Structural estimation of the model suggests network proximity of the second-order explains a dominant majority of trade frictions.

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A Appendix: Model

The model economy $\mathcal{E} \equiv \{\mathcal{M}, \mathcal{L}, \mathcal{J}\}$ consists of many firms ($\mathcal{M}$) and households ($\mathcal{L}$) at many locations ($\mathcal{J}$). Firms produce using local labor and intermediate inputs sourced from suppliers potentially spread across multiple locations. Each household supplies one unit of labor inelastically to local firms. Firms rebate any profits to local households. Trade between locations is subject to iceberg trade costs denoted by $\tau_{od} \geq 1$.

A.1 Household Preferences

The utility function for any household i at location d is defined over a discrete number of tasks (also indexed by $k \in \mathcal{K}_d(i) \equiv \{1, \cdots, K_d(i)\}$) as:

$$u_d(i) = \prod_{k \in \mathcal{K}_d(i)} q_d(i,k)^{1/K_d(i)},$$
$$q_d(i,k) = \sum_{s \in \mathcal{S}_d(i)} q_{od}(s,i,k),$$

where $q_d(i,k)$ is the quantity of goods consumed to fulfill need k and $\mathcal{S}_d(i)$ is the restricted set of suppliers that i encounters due to search frictions.

For task k, household i chooses the supplier that offers the cheapest price, that is,

$$s_d^*(i,k) = \arg \min_{s \in \mathcal{S}_d(i)} \left\{ \frac{c_o(s)\tau_{od}}{a_{od}(s,i,k)} \right\}, \tag{A.1}$$

The effective price faced by i for task k denoted by $p_d(i,k)$ is then given by

$$p_d(i,k) = \min_{s \in \mathcal{S}_d(i)} \left\{ \frac{c_o(s)\tau_{od}}{a_{od}(s,i,k)} \right\}. \tag{A.2}$$

Now, taking $\{p_d(i,k) : k \in \mathcal{K}_d(i)\}$ as given, the household's indirect utility function can be defined as:

$$V_d(i) = \max_{\{q_d(i,k): k \in \mathcal{K}_d(i)\}} \prod_{k \in \mathcal{K}_d(i)} q_d(i,k)^{1/K_d(i)} \tag{A.3}$$
$$\text{subject to} \sum_{k \in \mathcal{K}_d(i)} p_d(i,k) q_d(i,k) = w_d.$$

A.2 Technology and Market Structure

The production function for any firm b at location d is defined over labor and a discrete number of tasks (indexed by $k \in \mathcal{K}_d(b) \equiv \{1, \cdots, K_d(b)\}$) as:

$$y_d(b) = z_d(b) \left(\frac{l_d(b)}{1-\alpha_d} \right)^{1-\alpha_d} \left(\frac{\prod_{k \in \mathcal{K}_d(b)} m_d(b,k)^{1/K_d(b)}}{\alpha_d} \right)^{\alpha_d},$$

$$m_d(b,k) = \sum_{s \in \mathcal{S}_d(b)} m_{od}(s,b,k),$$

where $l_d(b)$ is the amount of labor input used by firm b, $m_d(b,k)$ is the quantity of materials utilized to accomplish task k, $z_d(b)$ is the idiosyncratic Hicks-neutral productivity with which firm b produces, and $\mathcal{S}_d(b)$ is the restricted set of suppliers that b encounters due to search frictions.

For task k, firm b chooses the supplier that offers the cheapest price, that is,

$$s_d^*(b,k) = \arg \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \right\}. \tag{A.4}$$

The effective price faced by b for task k, denoted by $p_d(b,k)$, is given by

$$p_d(b,k) = \min_{s \in \mathcal{S}_d(b)} \left\{ \frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \right\}. \tag{A.5}$$

Taking wage w_d and effective prices $\{p_d(b,k) : k \in \mathcal{K}\}$ as given, the firm's unit cost function can be defined as:

$$c_d(b) = \min_{\{l_d(b), \{m_d(b,k): k \in \mathcal{K}_d(b)\}\}} w_d l_d(b) + \sum_{k \in \mathcal{K}_d(b)} p_d(b,k)\, m_d(b,k) \tag{A.6}$$

$$\text{subject to } z_d(b) \left(\frac{l_d(b)}{1-\alpha_d} \right)^{1-\alpha_d} \left(\frac{\prod_{k \in \mathcal{K}_d(b)} m_d(b,k)^{1/K}}{\alpha_d} \right)^{\alpha_d} = 1.$$

A.3 Equilibrium Definition and Characterization

The aggregate state of the economy is denoted by $\sigma \equiv \{\boldsymbol{z}, \boldsymbol{\tau}, \boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{K}}, \boldsymbol{a}\}$ where

$$\boldsymbol{z} \equiv \{z_o(s) : s \in \mathcal{M}\},$$

$$
\begin{aligned}
\boldsymbol{\tau} &\equiv \left\{\tau_{od} : (o,d) \in \mathcal{J}^2\right\}, \\
\boldsymbol{\mathcal{S}} &\equiv \{\mathcal{S}_d(i) : i \in \mathcal{L} \cup \mathcal{M}\}, \\
\boldsymbol{\mathcal{K}} &\equiv \{\mathcal{K}_d(i) : i \in \mathcal{L} \cup \mathcal{M}\}, \text{ and} \\
\boldsymbol{a} &\equiv \{a_{od}(s,i,k) : (s,i,k) \in \mathcal{M} \times (\mathcal{L} \cup \mathcal{M}) \times \mathcal{K}\}
\end{aligned}
$$

An allocation in this economy is represented as $\xi \equiv \{\boldsymbol{l}(\sigma), \boldsymbol{m}(\sigma), \boldsymbol{q}(\sigma), \boldsymbol{y}(\sigma)\}$ and is defined as a set of functions,

$$
\begin{aligned}
\boldsymbol{l}(\sigma) &\equiv \{l_d(b;\sigma) : b \in \mathcal{M}\}, \\
\boldsymbol{m}(\sigma) &\equiv \left\{m_{od}(s,b,k;\sigma) : (s,b,k) \in \mathcal{M}^2 \times \mathcal{K}\right\}, \\
\boldsymbol{q}(\sigma) &\equiv \{q_{od}(s,i,k;\sigma) : (s,i,k) \in \mathcal{M} \times \mathcal{L} \times \mathcal{K}\}, \\
\boldsymbol{y}(\sigma) &\equiv \{y_o(s;\sigma) : s \in \mathcal{M}\},
\end{aligned}
$$

that map the realization of the state to intermediate input and labor quantities, quantities consumed and quantities produced. A price system is represented as $\varrho \equiv \{\boldsymbol{c}(\sigma), \boldsymbol{p}(\sigma), \boldsymbol{w}(\sigma)\}$ and is defined as a set of functions,

$$
\begin{aligned}
\boldsymbol{c}(\sigma) &\equiv \{c_o(s;\sigma) : s \in \mathcal{M}\}, \\
\boldsymbol{p}(\sigma) &\equiv \{p_d(i,k;\sigma) : (i,k) \in (\mathcal{L} \cup \mathcal{M}) \times \mathcal{K}\}, \\
\boldsymbol{w}(\sigma) &\equiv \{w_d(\sigma) : d \in \mathcal{J}\},
\end{aligned}
$$

that map the realization of the state to tasks' prices for firms, needs' prices for households, wage at each location and marginal costs of firms. This leads to the definition of equilibrium in this economy as follows.

Definition 3. For any given state σ, an equilibrium in this economy is defined as an allocation and price system, (ξ, ϱ) such that (a) households select suppliers for needs and firms select suppliers for tasks according to equations (3.1) and (A.1) respectively; (b) firms set prices for other firms and households according to equations (A.5) and (A.2) respectively; (c) households maximize utility according to equation (A.3); (d) firms minimize costs according to equation (A.6); and (e) market clears for each firm's goods and for labor at each location as follows.

$$
\sum_{b \in \mathcal{M}_d} l_d(b) = L_d
$$

$$\sum_{i\in\mathcal{L}}\sum_{k\in\mathcal{K}_d(i)} \frac{\tau_{od} q_d(i,k)}{a_{od}(s,i,k)} \mathbf{1}\left\{s = s_d^*(i,k)\right\}$$
$$+ \sum_{b\in\mathcal{M}}\sum_{k\in\mathcal{K}_d(i)} \frac{\tau_{od} m_d(b,k)}{a_{od}(s,b,k)} \mathbf{1}\left\{s = s_d^*(b,k)\right\} = y_o(s)$$

B Appendix: Estimation and Results

B.1 Proof of Proposition 1

Consider a pair of firms s located in o and b located in d. Now, suppose the marginal cost of firm s from o and it's cost of shipping goods to d are $c_o(s)$ and τ_{od} respectively. For any task k and match-specific productivity $a_{od}(s,b,k) = a$, the effective cost incurred by s of delivering its goods for task k by b is $\frac{c_o(s)\tau_{od}}{a}$. Supplier s is selected by b for task k if b encounters s with match-specific productivity a and b does not encounter any other supplier for whom it is effectively less costly to deliver the good (including the event that b meets s and the match-specific productivity realized is higher than a). The probability with which b selects s for any of its tasks with match-specific productivity a is given by:

$$\begin{aligned}
\pi_{od}^0(s,b,k \mid a) &= \frac{\lambda\phi_{od}(s)}{M} \times \prod_{s'\in\mathcal{M}} \left(1 - \frac{\lambda\phi_{o'd}(s')}{M} \mathbb{I}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a}\right)\right) \\
&= \frac{\lambda\phi_{od}(s)}{M} \times \exp\left(\sum_{s'\in\mathcal{M}} \ln\left(1 - \frac{\lambda\phi_{o'd}(s')}{M} \mathbb{P}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a}\right)\right)\right)
\end{aligned}$$

Since $\lambda = o(M)$, considering $\frac{\lambda}{M} \ll 1$ and using the approximation $\ln(1+x) \approx x$ for $|x| \ll 1$, the above expression simplifies as:

$$\pi_{od}^0(s,b,k \mid a) = \frac{\lambda\phi_{od}(s)}{M} \exp\left(- \frac{\lambda}{M} \sum_{s'\in\mathcal{M}} \phi_{o'd}(s') \mathbb{P}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a}\right)\right.$$

Taking expectation over all possible realizations of $a_{od}(s,b,k)$, we obtain:

$$\begin{aligned}
\pi_{od}^0(s,b,k) &= \mathbb{E}_{\{a\}}\left[\pi_{od}^0(s,b,k \mid a)\right] \\
&= \frac{\lambda\phi_{od}(s)}{M} \int_0^\infty \exp\left(-\frac{\lambda}{M} \sum_{s'\in\mathcal{M}} \phi_{o'd}(s') \mathbb{P}\left(\frac{c_{o'}(s')\tau_{o'd}}{a_{o'd}(s',b,k)} \leq \frac{c_o(s)\tau_{od}}{a}\right)\right) dF_a(a)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\lambda \phi_{od}(s)}{M} \int_{a_0}^{\infty} \exp\left(-\frac{\lambda}{M} \sum_{s' \in \mathcal{M}} \phi_{o'd}(s') \mathbb{P}\left(a_{o'd}(s', b, k) \geq \frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}} a \right) \right) d\left(1 - (a/a_0)^{-\zeta}\right) \\
&= \frac{\lambda a_0^{\zeta} \phi_{od}(s)}{M} \int_{a_0}^{\infty} \exp\Bigg(-\frac{\lambda a_0^{\zeta}}{M} \sum_{s' \in \mathcal{M}} \phi_{o'd}(s') \mathbf{1}\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}} a \geq a_0 \right) \left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}} a \right)^{-\zeta} \\
&- \frac{\lambda}{M} \sum_{s' \in \mathcal{M}} \phi_{o'd}(s') \mathbf{1}\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}} a \leq a_0 \right) \Bigg) \zeta a^{-\zeta-1} da \\
&= \frac{\phi_{od}(s)}{M} \int_{0}^{\infty} \exp\left(-\frac{1}{M} \sum_{s' \in \mathcal{M}} \phi_{o'd}(s') \left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}} \right)^{-\zeta} a^{-\zeta} \right) d\left(-a^{-\zeta}\right) \\
&= \frac{c_o(s)^{-\zeta} \phi_{od}(s) \tau_{od}^{-\zeta}}{\sum_{s' \in \mathcal{M}} c_{o'}(s')^{-\zeta} \phi_{o'd}(s') \tau_{o'd}^{-\zeta}} \Gamma(1) \\
&= \frac{c_o(s)^{-\zeta} \phi_{od}(s) \tau_{od}^{-\zeta}}{\sum_{s' \in \mathcal{M}} c_{o'}(s')^{-\zeta} \phi_{o'd}(s') \tau_{o'd}^{-\zeta}}
\end{aligned}
$$

Here, in the fifth line we utilize Assumption 2 which implies that in sufficiently large economies $\lim_{t\to\infty} \lambda_t a_{0,t}^{\zeta} \to 1$ and $\lim_{t\to\infty} a_{0,t} \to 0$ such that $\frac{\lambda}{M} \sum_{s' \in \mathcal{M}} \phi_{o'd}(s') \mathbf{1}\left(\frac{c_{o'}(s')\tau_{o'd}}{c_o(s)\tau_{od}} a \leq a_0 \right) \to 0$ for all firms s'. Since $\pi_{od}(s, b, k)$ is independent of the identity of the task k, we write $\pi_{od}^0(s, b) = \pi_{od}^0(s, b, k)$. Further, since $\pi_{od}^0(s, b)$ is independent of the identity of the buyer at any location d, we write $\pi_{od}^0(s, -) = \pi_{od}^0(s, b)$.

B.2 Proof of Proposition 2

In our context, the multinomial random variable counts the number of successes in each of the M categories (one for each other supplier s), after $K_d(b)$ independent trials (one for each task associated with b). Let $\pi_{od}^0(s, b)$ denote the probability of success and $K_{od}(s, b)$ denote the number of successes in category s, the probability of observing $\{K_{od}(s, b) : s \in \mathcal{M}_o, o \in \mathcal{J}\}$ conditional on the number of tasks $K_d(b)$ is:

$$
\mathbb{P}\left(\{K_{od}(s, b) : s \in \mathcal{M}\} \mid K_d(b) \right) = K_d(b)! \prod_{s \in \mathcal{M}} \frac{\left(\pi_{od}^0(s, b)\right)^{K_{od}(s,b)}}{K_{od}(s, b)!}
$$

where $\sum_{s \in \mathcal{M}} \pi_{od}^0(s, b) = 1$ and $\sum_{o \in \mathcal{J}} \sum_{s \in \mathcal{M}_o} K_{od}(s, b) = K_d(b)$. The likelihood for the complete sample, $\mathbb{K} \equiv \left\{K_{od}(s, b) : (s, b) \in \mathcal{M}^2\right\}$ with probabilities $\mathbf{\Pi}^0 \equiv \left\{\pi_{od}^0(s, b) : (s, b) \in \mathcal{M}^2\right\}$ scaled by a factor $K_d(b)$ for each firm b is:

$$
\begin{aligned}
\ell\left(\mathbb{K} \mid \mathbf{\Pi}^0, \mathcal{K}\right) &= \prod_{b\in\mathcal{M}} \left(K_d(b)! \prod_{s\in\mathcal{M}} \frac{\left(\pi^0_{od}(s,b)\right)^{K_{od}(s,b)}}{K_{od}(s,b)!} \right)^{\frac{1}{K_d(b)}} \\
&= \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{K_d(b)!}{K_{od}(s,b)!} \left(\pi^0_{od}(s,b)\right)^{\frac{K_{od}(s,b)}{K_d(b)}} \right) \\
&= \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{K_d(b)!}{K_{od}(s,b)!} \left(\pi^0_{od}(s,b)\right)^{\sum_{k\in\mathcal{K}_d(b)} \mathbb{I}\left\{s=s^*_d(b,k)\right\} \frac{1}{K_d(b)}} \right) \\
&= \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{K_d(b)!}{K_{od}(s,b)!} \left(\pi^0_{od}(s,b)\right)^{\sum_{k\in\mathcal{K}_d(b)} \mathbb{I}\left\{s=s^*_d(b,k)\right\} \frac{\text{purchases}_d(b,k)}{\text{purchases}_d(b)}} \right) \\
&= \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{K_d(b)!}{K_{od}(s,b)!} \left(\pi^0_{od}(s,b)\right)^{\frac{\sum_{k\in\mathcal{K}_d(b)} \mathbb{I}\left\{s=s^*_d(b,k)\right\} \text{purchases}_d(b,k)}{\text{purchases}_d(b)}} \right) \\
&= \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{K_d(b)!}{K_{od}(s,b)!} \left(\pi^0_{od}(s,b)\right)^{\frac{\text{sales}_{od}(s,b)}{\text{purchases}_d(b)}} \right) \\
&= \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{K_d(b)!}{K_{od}(s,b)!} \left(\pi^0_{od}(s,b)\right)^{\pi_{od}(s,b)} \right) \\
&= \prod_{b\in\mathcal{M}} \left(\prod_{s\in\mathcal{M}} \frac{K_d(b)!}{K_{od}(s,b)!} \left(\frac{c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta}} \right)^{\pi_{od}(s,b)} \right)
\end{aligned}
$$

Therefore, the log-likelihood is proportional to:

$$
\begin{aligned}
\mathcal{L}\left(\mathbb{K} \mid \mathbf{\Pi}^0, \mathcal{K}\right) \propto& \left(\sum_{b\in\mathcal{M}} \sum_{s\in\mathcal{M}} \pi_{od}(s,b) \ln\left(c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}\right) \right) \\
&- \left(\sum_{b\in\mathcal{M}} \sum_{s\in\mathcal{M}} \pi_{od}(s,b) \ln\left(\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta} \right) \right) \\
=& \sum_{d\in\mathcal{J}} \left(\sum_{s\in\mathcal{M}} \left(\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b) \right) \ln\left(c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}\right) \right) \\
&- \left(\sum_{b\in\mathcal{M}} \left(\sum_{s\in\mathcal{M}} \pi_{od}(s,b) \right) \ln\left(\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta} \right) \right) \\
=& \sum_{d\in\mathcal{J}} \left(\sum_{s\in\mathcal{M}} \left(\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b) \right) \ln\left(c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}\right) \right)
\end{aligned}
$$

$$- \sum_{d \in \mathcal{J}} M_d \ln \left(\sum_{s' \in \mathcal{M}} c_{o'}(s')^{-\zeta} \phi_{o'd}(s') \tau_{o'd}^{-\zeta} \right)$$

Note that for all $s \in \mathcal{M}$, $c_o(s) = c_o \widetilde{c}_o(s)$ and $\phi_{od}(s) = \overline{\phi_{od}} \widetilde{\phi}_{od}(s)$ such that

$$\left(\sum_{s \in \mathcal{M}_o} \widetilde{c}_o(s)^{-\zeta} \widetilde{\phi}_{od}(s) \right)^{-1/\zeta} = 1, \tag{B.1}$$

$$\sum_{d \in \mathcal{J}} \frac{\frac{c_o^{-\zeta} \overline{\phi_{od}} \tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta} \overline{\phi_{o'd}} \tau_{o'd}^{-\zeta}} M_d}{\sum_{d \in \mathcal{J}} \frac{c_o^{-\zeta} \overline{\phi_{od}} \tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta} \overline{\phi_{o'd}} \tau_{o'd}^{-\zeta}} M_d} \widetilde{\phi}_{od}(s) = 1. \tag{B.2}$$

The likelihood equations for $\{\widetilde{c}_o(s) : s \in \mathcal{M}\}$ are given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}\left(\mathbb{K} \mid \mathbf{\Pi}^0, \boldsymbol{\mathcal{K}}\right)}{\partial \widetilde{c}_o(s)^{-\zeta}} &= 0 \\
\implies \left(\sum_{d \in \mathcal{J}} \left(\sum_{b \in \mathcal{M}_d} \pi_{od}(s,b) \right) \frac{1}{\widetilde{c}_o(s)^{-\zeta}} \right) & \\
- \sum_{d \in \mathcal{J}} M_d \frac{c_o^{-\zeta} \phi_{od}(s) \tau_{od}^{-\zeta}}{\sum_{s' \in \mathcal{M}} c_{o'}(s')^{-\zeta} \phi_{o'd}(s') \tau_{o'd}^{-\zeta}} &= 0 \\
\implies \left(\sum_{d \in \mathcal{J}} \sum_{b \in \mathcal{M}_d} \pi_{od}(s,b) \right) \frac{1}{\widetilde{c}_o(s)^{-\zeta}} & \\
- \sum_{d \in \mathcal{J}} M_d \widetilde{\phi}_{od}(s) \frac{c_o^{-\zeta} \overline{\phi_{od}} \tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta} \overline{\phi_{o'd}} \tau_{o'd}^{-\zeta}} &= 0 \\
\implies \sum_{d \in \mathcal{J}} M_d \frac{c_o(s)^{-\zeta} \phi_{od}(s) \tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta} \overline{\phi_{o'd}} \tau_{o'd}^{-\zeta}} &= \sum_{d \in \mathcal{J}} \sum_{b \in \mathcal{M}_d} \pi_{od}(s,b) \\
\implies \sum_{d \in \mathcal{J}} M_d \widetilde{c}_o(s)^{-\zeta} \widetilde{\phi}_{od}(s) \frac{c_o^{-\zeta} \overline{\phi_{od}} \tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta} \overline{\phi_{o'd}} \tau_{o'd}^{-\zeta}} &= \sum_{d \in \mathcal{J}} \sum_{b \in \mathcal{M}_d} \pi_{od}(s,b) \qquad \text{(B.3)}
\end{aligned}$$

The likelihood equations for $\left\{\widetilde{\phi}_{od}(s) : (s,d) \in \mathcal{M} \times \mathcal{J}\right\}$ are given by:

$$\frac{\partial \mathcal{L}\left(\mathbb{K} \mid \mathbf{\Pi}^0, \boldsymbol{\mathcal{K}}\right)}{\partial \widetilde{\phi}_{od}(s)} = 0$$

$$
\begin{aligned}
&\implies \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)\frac{1}{\widetilde{\phi}_{od}(s)} \\
&-M_d\frac{c_o(s)^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta}} = 0 \\
&\implies M_d\frac{c_o(s)^{-\zeta}\overline{\phi_{od}}\widetilde{\phi}_{od}(s)\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} = \sum_{b\in\mathcal{M}_d}\pi_{od}(s,b) \\
&\implies \widetilde{c}_o(s)^{-\zeta}\widetilde{\phi}_{od}(s)\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} = \frac{1}{M_d}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)
\end{aligned}
\tag{B.4}
$$

The likelihood equations for $\left\{\overline{\phi_{od}}\tau_{od}^{-\zeta} : (o,d)\in\mathcal{J}^2\right\}$ are given by:

$$
\begin{aligned}
&\frac{\partial\mathcal{L}\left(\mathbb{K}\mid\mathbf{\Pi}^0,\mathcal{K}\right)}{\partial\left(\overline{\phi_{od}}\tau_{od}^{-\zeta}\right)} = 0 \\
&\implies \sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)\frac{1}{\overline{\phi_{od}}\tau_{od}^{-\zeta}} \\
&-M_d\sum_{s\in\mathcal{M}_o}\frac{c_o(s)^{-\zeta}\widetilde{\phi}_{od}(s)}{\sum_{s'\in\mathcal{M}} c_{o'}(s')^{-\zeta}\phi_{o'd}(s')\tau_{o'd}^{-\zeta}} = 0 \\
&\implies M_d\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} = \sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b) \\
&\implies \frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} = \frac{1}{M_d}\sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)
\end{aligned}
\tag{B.5}
$$

Dividing equation (B.4) by equation (B.5), we obtain:

$$
\begin{aligned}
&\frac{\widetilde{c}_o(s)^{-\zeta}\widetilde{\phi}_{od}(s)\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}}}{\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}}} = \frac{\frac{1}{M_d}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)}{\frac{1}{M_d}\sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)} \\
&\implies \widetilde{c}_o(s)^{-\zeta}\widetilde{\phi}_{od}(s) = \frac{\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)}{\sum_{s\in\mathcal{M}_o}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)}
\end{aligned}
\tag{B.6}
$$

Substituting equations (B.2) and (B.5) in equation (B.3), we obtain:

$$
\sum_{d\in\mathcal{J}} M_d\widetilde{c}_o(s)^{-\zeta}\widetilde{\phi}_{od}(s)\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} = \sum_{d\in\mathcal{J}}\sum_{b\in\mathcal{M}_d}\pi_{od}(s,b)
$$

$$\Longrightarrow \widetilde{c}_o(s)^{-\zeta} \sum_{d\in\mathcal{J}} \widetilde{\phi}_{od}(s) \frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} M_d = \sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)$$

$$\Longrightarrow \widetilde{c}_o(s)^{-\zeta} \sum_{d\in\mathcal{J}} \frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}} M_d = \sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)$$

$$\Longrightarrow \widetilde{c}_o(s)^{-\zeta} \sum_{s\in\mathcal{M}_o} \sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b) = \sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)$$

$$\Longrightarrow \widetilde{c}_o(s)^{-\zeta} = \frac{\sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{\sum_{s\in\mathcal{M}_o} \sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)} \tag{B.7}$$

Dividing equation (B.6) by equation (B.7), we obtain:

$$\widetilde{\phi}_{od}(s) = \frac{\frac{\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{\sum_{s'\in\mathcal{M}_o} \sum_{b\in\mathcal{M}_d} \pi_{od}(s',b)}}{\frac{\sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{\sum_{s'\in\mathcal{M}_o} \sum_{d\in\mathcal{J}} \sum_{b\in\mathcal{M}_d} \pi_{od}(s',b)}}$$

Putting these together and simplifying, we obtain:

$$\left(\widetilde{c}_o(s)^{-\zeta}\right)^* = \frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{\sum_{b\in\mathcal{M}} \pi_{od}(\bullet,b)}$$

$$\left(\widetilde{\phi}_{od}(s)\right)^* = \frac{\frac{\sum_{b\in\mathcal{M}_d} \pi_{od}(s,b)}{\sum_{b\in\mathcal{M}_d} \pi_{od}(\bullet,b)}}{\frac{\sum_{b\in\mathcal{M}} \pi_{od}(s,b)}{\sum_{b\in\mathcal{M}} \pi_{od}(\bullet,b)}}$$

$$\left(\frac{c_o^{-\zeta}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{\sum_{o'} c_{o'}^{-\zeta}\overline{\phi_{o'd}}\tau_{o'd}^{-\zeta}}\right)^* = \frac{1}{M_d} \sum_{b\in\mathcal{M}_d} \pi_{od}(\bullet,b)$$

C Appendix: Aggregation

C.1 Continuum Approximation for Large Network Economies

The following definition formalizes the notion of the limiting economy in the context of this paper.

Definition 4. Consider a sequence of finite economies $\{\mathcal{E}_t : t \in \mathbb{N}\}$ where $\mathcal{E}_t \equiv \{\mathcal{M}_t, \mathcal{L}_t, \mathcal{J}_t\}$ is such that the t^{th} economy has the form $\mathcal{M}_t = \{m_1, \cdots, m_{M_t}\} \subset [0,1]$, $\mathcal{L}_t = \{\ell_1, \cdots, \ell_{L_t}\} \subset [0,1]$ and $\mathcal{J}_t = \mathcal{J}$. The uniform distribution on $\mathcal{M}_t$ is given by $\mathcal{U}_t^M\left(\mathcal{M}_t^0\right) = \frac{M_t^0}{M_t}$ for all $\mathcal{M}_t^0 \subset \mathcal{M}_t$. Similarly, the uniform distribution on $\mathcal{L}_t$ is given by $\mathcal{U}_t^L\left(\mathcal{L}_t^0\right) = \frac{L_t^0}{L_t}$ for all $\mathcal{L}_t^0 \subset \mathcal{L}_t$. Then, $\{\mathcal{E}_t : t \in \mathbb{N}\}$ is a discretizing sequence of economies if it satisfies:

1. $\mathcal{M}_t \subset \mathcal{M}_{t+1}$ and $\mathcal{L}_t \subset \mathcal{L}_{t+1}$ for all t,
2. $\lim_{t\to\infty} \mathcal{U}_t^M \left(\mathcal{M}_t \cap [a_l, a_h]\right) = \mathcal{U}\left([a_l, a_h]\right)$,
3. $\lim_{t\to\infty} \mathcal{U}_t^L \left(\mathcal{L}_t \cap [a_l, a_h]\right) = \mathcal{U}\left([a_l, a_h]\right)$,

where $\mathcal{U}(\bullet)$ denotes the uniform distribution with support over $[0, 1]$ and $[a_l, a_h] \subset [0, 1]$.

Assumption 2. *The discretizing sequence of economies $\{\mathcal{E}_t : t \in \mathbb{N}\}$ satisfies the following conditions:*[5]

1. $\{\lambda_t, a_{0,t} : t \in \mathbb{N}\}$ is such that $\lambda_t = o(M_t)$ and $\lambda_t a_{0,t}^{\zeta} = \Theta(1)$
2. $\{M_{d,t}, L_{d,t} : d \in \mathcal{J}, t \in \mathbb{N}\}$ is such that $M_{d,t} = \Theta(M_t)$ and $L_{d,t} = \Theta(L_t)$ for all $d \in \mathcal{J}$

C.2 Proof of Proposition 3

C.2.1 Distribution of the Lowest Effective Cost

We begin by characterizing the distribution of the lowest effective cost available to buyer b located at d, $F_{p_d}(p) = \mathbb{P}\left(p_d^*(b,k) \le p\right)$. It is convenient to think about the complementary probability $\mathbb{P}\left(p_d^*(b,k) \ge p\right)$, the probability that the lowest effective price faced by the buyer is no less than p. To do so, we evaluate the probability with which b receives no offers less than than p. The lowest cost offer p can be from any one of the locations in $\mathcal{J}$. We evaluate the probability with which this offer is from any given location o and multiply it across all locations. The probability with which b receives one offer with an effective cost no less than p from o:

$$\begin{cases} \prod_{s\in\mathcal{M}_o}\left(1 - \frac{\lambda\phi_{od}(s)}{M}\mathbb{I}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \le p\right)\right) & \text{if } o \neq d \\ \prod_{s\in\mathcal{M}_o\setminus\{b\}}\left(1 - \frac{\lambda\phi_{od}(s)}{M}\mathbb{I}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \le p\right)\right) & \text{if } o = d \end{cases}$$

Under Assumption 2, the probability with which b encounters no supplier who can deliver at a cost no less than p across all locations is given by:

$$F_{p_d}(p) = 1 - \prod_{s\in\mathcal{M}\setminus\{b\}}\left(1 - \frac{\lambda\phi_{od}(s)}{M}\mathbb{I}\left(\frac{c_o(s)\tau_{od}}{a_{od}(s,b,k)} \le p\right)\right)$$

[5] For any two functions $f(n)$ and $g(n)$, $f(n) = o(g(n)) \implies \lim_{n\to\infty}\frac{f(n)}{g(n)} = 0$ and $f(n) = \Theta(g(n) \implies \limsup_{n\to\infty}\frac{|f(n)|}{g(n)} < \infty$ and $\limsup_{n\to\infty} \mid \frac{f(n)}{g(n)} \mid > 0$.

$$= 1 - \exp\left(-\sum_o \lambda \mu_o \mathbb{E}\left[\phi_{od}(\cdot)\right] \mathbb{P}\left(\frac{c_o(\cdot)\tau_{od}}{a_{od}(\cdot, b, k)} \leq p\right)\right)$$

Using the limit $\lim_{t\to\infty} \lambda_t a_{0,t}^{\zeta} \to 1$, this can be further simplified as $1 - \exp\left(-A_d p^{\zeta}\right)$ where $A_d = \sum_o \mu_o \tau_{od}^{-\zeta} \mathbb{E}\left[\phi_{od}(\cdot)\right] \mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]$ is obtained as follows:

$$\begin{aligned}
A_d p^{\zeta} &= \sum_o \lambda \mu_o \mathbb{E}\left[\phi_{od}(\cdot)\right] \mathbb{P}\left(\frac{c_o(\cdot)\tau_{od}}{a_{od}(\cdot, b, k)} \leq p\right) \\
&= \sum_o \lambda \mu_o \mathbb{E}\left[\phi_{od}(\cdot)\right] \mathbb{E}\left[1 - F_a\left(\frac{c_o(\cdot)\tau_{od}}{p}\right)\right] \\
&= \left(\sum_o \mu_o \tau_{od}^{-\zeta} \mathbb{E}\left[\phi_{od}(\cdot)\right] \mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]\right) p^{\zeta} \\
\implies A_d &= \sum_o \mu_o \tau_{od}^{-\zeta} \mathbb{E}\left[\phi_{od}(\cdot)\right] \mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]
\end{aligned}$$

C.2.2 Derivation of Market Access

$$\begin{aligned}
c_o(\cdot) &= w_o^{1-\alpha_o} \left(\prod_{k \in \mathcal{K}_o(\cdot)} p_o(\cdot, k)^{1/K_o(\cdot)}\right)^{\alpha_o} \\
\implies \mathbb{E}\left[c_o(\cdot)^{-\zeta}\right] &= \mathbb{E}\left[\left(\frac{w_o^{1-\alpha_o}\left(\prod_{k \in \mathcal{K}_o(\cdot)} p_o(\cdot, k)^{1/K_o(\cdot)}\right)^{\alpha_o}}{z_o(\cdot)}\right)^{-\zeta}\right] \\
&= w_o^{-\zeta(1-\alpha_o)} \mathbb{E}\left[\prod_{k \in \mathcal{K}_o(\cdot)} p_o(\cdot, k)^{-\alpha_o \zeta / K_o(\cdot)}\right] \mathbb{E}\left[z_o(\cdot)^{\zeta}\right] \\
&= w_o^{-\zeta(1-\alpha_o)} \left(\mathbb{E}\left[\mathbb{E}\left[\prod_{k \in \mathcal{K}_o(\cdot)} p_o(\cdot, k)^{-\alpha_o \zeta / K_o(\cdot)} \mid K_o\right]\right]\right) \mathbb{E}\left[z_o(\cdot)^{\zeta}\right] \\
&= w_o^{-\zeta(1-\alpha_o)} \left(\mathbb{E}\left[\prod_{k \in \mathcal{K}_o(\cdot)} \Gamma\left(1 - \frac{\alpha_o}{K_o(\cdot)}\right) A_o^{\frac{\alpha_o}{K_o(\cdot)}}\right]\right) \overline{z_o^{\zeta}} \\
&= \mathbb{E}\left[\Gamma\left(1 - \frac{\alpha_o}{K_o(\cdot)}\right)^{K_o(\cdot)}\right] \overline{z_o^{\zeta}} w_o^{-\zeta(1-\alpha_o)} A_o^{\alpha_o}
\end{aligned}$$

This implies that $\{A_d\}_{d\in\mathcal{J}}$ solves the following fixed point problem:

$$A_d = \sum_o \mu_o \mathbb{E}\left[\phi_{od}(\cdot)\right] \tau_{od}^{-\zeta} \overline{z_o^{\zeta}} \mathbb{E}\left[\Gamma\left(1-\frac{\alpha_o}{K_o(\cdot)}\right)^{K_o(\cdot)}\right] w_o^{-\zeta(1-\alpha_o)} A_o^{\alpha_o}$$

It can be similarly shown that effective prices for needs faced by households is also given by $F_{p_d}(\cdot)$ The following lemma states that the above fixed point problem that solves for market access is well-defined in the sense that it admits a unique positive solution. The proof strategy follows from Allen et al. (2020).

Lemma. *The following system of equations*

$$A_d = \sum_o R_{od} A_o^{\alpha_o},$$
$$R_{od} = \mu_o \mathbb{E}\left[\phi_{od}(\cdot)\right] \tau_{od}^{-\zeta} \overline{z_o^{\zeta}} \mathbb{E}\left[\Gamma\left(1-\frac{\alpha_o}{K_o(\cdot)}\right)^{K_o(\cdot)}\right] w_o^{-\zeta(1-\alpha_o)} A_o^{\alpha_o}.$$

1. *has at least one positive solution*

2. *has at most one positive solution (up to scale)*

3. *the unique solution can be computed as the limit of a simple iterative procedure.*

Proof. First, I establish existence of positive solution to the system of equations. Define operator $T : \mathbb{R}_{++}^J \to \mathbb{R}_{++}^J$ where $T(\boldsymbol{A}) = \left(\sum_o R_{o1} A_o^{\alpha_o}, \cdots, \sum_o R_{oJ} A_o^{\alpha_o}\right)'$. Note that all components of R_{od} are positive and finite. Then, by construction, for any d, not all R_{od}s are zero. Therefore, for any $\boldsymbol{A} \gg 0$, $\sum_o R_{o1} A_o^{\alpha_o} \geq \underline{A} > 0$. Further, there exists $\bar{A} < \infty$ such that $\sum_o R_{od} A_o^{\alpha_o} \leq \bar{A}$. Now consider the operator $T : \mathcal{A} \to \mathcal{A}$ defined by $T(A_1, \cdots, A_J) = \left(\sum_o R_{o1} A_o^{\alpha_o}, \cdots, \sum_o R_{oJ} A_o^{\alpha_o}\right)'$. Suppose $\mathcal{A} = \left\{\boldsymbol{A} \in \mathbb{R}_{++}^J \mid \underline{A} \leq A_d \leq \bar{A} \forall d\right\}$.Then, if $\boldsymbol{A} \gg 0$, it follows that $T(\boldsymbol{A}) \gg 0$. Note that $\mathcal{A}$ is closed and bounded. Since $\mathcal{A} \subset \mathbb{R}_{++}^J$, this implies that $\mathcal{A}$ is compact. Further, $\mathcal{A}$ is non-empty and convex, and T is continuous. Then, by Brouwer's fixed point theorem, $T(\bullet)$ has a fixed point. This establishes existence of a solution the system of equations.

To establish uniqueness, let's suppose by way of contradiction that the system of equations has two different solutions $\boldsymbol{A}^{(0)}, \boldsymbol{A}^{(1)}$ that are not linear transformations of each other. Denote $\bar{a} = \max_d \frac{A_d^{(1)}}{A_d^{(0)}}$ and $\underline{a} = \min_d \frac{A_d^{(1)}}{A_d^{(0)}}$. Notice that $\frac{\bar{a}}{\underline{a}} \geq 1$. Thus the system of equations can be expressed as:

$$\frac{A_d^{(1)}}{A_d^{(0)}} = \frac{\sum_o R_{od} \left(\frac{A_d^{(1)}}{A_d^{(0)}} \right)^{1-\alpha_o} \left(A_d^{(0)} \right)^{1-\alpha_o}}{A_d^{(0)}}$$

Suppose $\bar{d} = \arg\max_d \left(\frac{A_d^{(1)}}{A_d^{(0)}} \right)$ and $\underline{\alpha} = \min \alpha_o$, then we have:

$$\begin{aligned} \frac{A_{\bar{d}}^{(1)}}{A_{\bar{d}}^{(0)}} &= \bar{a} \\ \implies \frac{\sum_o R_{o\bar{d}} \left(\frac{A_o^{(1)}}{A_o^{(0)}} \right)^{1-\alpha_o} \left(A_o^{(0)} \right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} &= \bar{a} \\ \implies \frac{\sum_o R_{o\bar{d}} \bar{a}^{1-\underline{\alpha}} \left(A_o^{(0)} \right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} &\geq M \\ \implies \frac{\sum_o R_{o\bar{d}} \left(A_o^{(0)} \right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} \bar{a}^{1-\underline{\alpha}} &\geq \bar{a} \\ \implies \bar{a}^{\underline{\alpha}} &\leq 1 \\ \implies \bar{a} &\leq 1 \end{aligned}$$

Similarly, we can show that $\underline{a} \geq 1$. This implies that $\frac{\bar{a}}{\underline{a}} \leq 1$. But by construction $\frac{\bar{a}}{\underline{a}} \geq 1$. Therefore, it must be the case that $\frac{\bar{a}}{\underline{a}} = 1$ or $\boldsymbol{A}^{(0)} = \boldsymbol{A}^{(1)}$. This establishes uniqueness.

Next, I show that the solution to the system of equations can be obtained via a simple iterative procedure. Starting from any strictly positive $\boldsymbol{A}^{(0)}$, we construct a sequence $\boldsymbol{A}^{(t)}$ successively in the following way,

$$A_d^{(t)} = \sum_o R_{od} \left(A_o^{(t-1)} \right)^{\alpha_o}$$

Denote $\bar{a}^{(t)} = \max_d \frac{A_d^{(t)}}{A_d^{(t-1)}}$ and $\underline{a}^{(t)} = \min_d \frac{A_d^{(t)}}{A_d^{(t-1)}}$. Notice that $\frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} \geq 1$.

Suppose $\bar{d} = \arg\max_d \left(\frac{A_d^{(t)}}{A_d^{(t-1)}} \right)$ and $\underline{\alpha} = \min \alpha_o$, then we have:

$$
\begin{aligned}
\frac{A_{\bar{d}}^{(t)}}{A_{\bar{d}}^{(t-1)}} &= \bar{a}^{(t)} \\
\implies \frac{\sum_o R_{o\bar{d}} \left(\frac{A_o^{(t-1)}}{A_o^{(t-2)}}\right)^{1-\alpha_o} \left(A_o^{(t-2)}\right)^{1-\alpha_o}}{A_{\bar{d}}^{(t-1)}} &= \bar{a}^{(t)} \\
\implies \frac{\sum_o R_{o\bar{d}} \left(A_o^{(0)}\right)^{1-\alpha_o}}{A_{\bar{d}}^{(0)}} \left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}} &\geq \bar{a}^{(t)} \\
\implies \frac{\bar{a}^{(t)}}{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}} &\leq 1
\end{aligned}
$$

Similarly, we can show that $\frac{\underline{a}^{(t)}}{\left(\underline{a}^{(t-1)}\right)^{1-\bar{\alpha}}} \geq 1$. This implies the following

$$
\begin{aligned}
\frac{\bar{a}^{(t)}}{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}} &\leq \frac{\underline{a}^{(t)}}{\left(\underline{a}^{(t-1)}\right)^{1-\bar{\alpha}}} \\
\implies \frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} &\leq \frac{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}}{\left(\underline{a}^{(t-1)}\right)^{1-\bar{\alpha}}} \\
&\leq \frac{\left(\bar{a}^{(t-1)}\right)^{1-\underline{\alpha}}}{\left(\underline{a}^{(t-1)}\right)^{1-\underline{\alpha}}} \\
\implies \frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} &\leq \frac{\bar{a}^{(t-1)}}{\underline{a}^{(t-1)}}
\end{aligned}
$$

Since $\frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} \geq 1 \forall t$, this implies that $\lim_{t\to\infty} \frac{\bar{a}^{(t)}}{\underline{a}^{(t)}} = 1$. That is, the solution can be computed as the limit of a simple iterative procedure. □

C.3 Proof of Proposition 4

The probability with which any firm at d sources from firms at o for any of its tasks is given by

$$
\pi_{od}^0 (\bullet, -) = \left(\lim_{t\to\infty} \frac{M_o}{M}\right) \left(\lim_{t\to\infty} \frac{1}{M_o} \sum_{s\in\mathcal{M}_o} \pi_{od}^0(s, -)\right)
$$

$$
\begin{aligned}
&= \left(\lim_{t\to\infty} \frac{M_o}{M}\right)\left(\lim_{t\to\infty} \frac{1}{M_o} \sum_{s\in\mathcal{M}_o} \frac{c_o(s)^{-\zeta}\phi_{od}(s)\tau_{od}^{-\zeta}}{A_d}\right) \\
&= \frac{\mu_o \mathbb{E}\left[\phi_{od}(\cdot)\right]\mathbb{E}\left[c_o(\cdot)^{-\zeta}\right]\tau_{od}^{-\zeta}}{A_d} \\
&= \frac{\mu_o \overline{z_o^{\zeta}} w_o^{-\zeta(1-\alpha_o)} \mathbb{E}\left[\Gamma\left(1-\frac{\alpha_o}{K_o(\cdot)}\right)^{K_o(\cdot)}\right] A_o^{\alpha_o}\overline{\phi_{od}}\tau_{od}^{-\zeta}}{A_d}
\end{aligned}
$$

C.4 Proof of Proposition 5

For any realization of σ, labor demand by firm b at d can be expressed as:

$$
l_d(b,\sigma) = \frac{1}{w_d(\sigma)}(1-\alpha_d)\, c_d(b,\sigma) y_d(b,\sigma)
$$

Substituting the above expression in the labor market clearing for location d, we obtain:

$$
\begin{aligned}
L_d &= \sum_{b\in\mathcal{M}_d} l_d(b,\sigma) \\
&= \sum_{b\in\mathcal{M}_d} \frac{1}{w_d(\sigma)}(1-\alpha_d)\, c_d(b,\sigma) y_d(b,\sigma) \\
\implies \sum_{b\in\mathcal{M}_d} c_d(b,\sigma) y_d(b,\sigma) &= \frac{w_d(\sigma) L_d}{1-\alpha_d}
\end{aligned}
$$

Goods market clearing condition for firm s located at o can be simplified as:

$$
\begin{aligned}
y_o(s,\sigma) &= \sum_d \sum_{b\in\mathcal{M}_d} \sum_{k\in\mathcal{K}_d(b)} \frac{\tau_{od}(s,\sigma) m_{od}(s,b,k,\sigma)}{a_{od}(s,b,k,\sigma)} \\
&\quad + \sum_d \sum_{i\in\mathcal{L}_d} \sum_{n\in\mathcal{N}_d(i)} \frac{\tau_{od}(s,\sigma) q_{od}(s,i,n,\sigma)}{g_{od}(s,i,n,\sigma)} \\
\implies c_o(s,\sigma) y_o(s,\sigma) &= \sum_d \alpha_d \sum_{b\in\mathcal{M}_d} \left(\frac{1}{K_d(b)} \sum_{k\in\mathcal{K}_d(b)} \mathbf{1}\left\{s = s_d^*(b,k,\sigma)\right\}\right) c_d(b,\sigma) y_d(b,\sigma) \\
&\quad + \sum_d \sum_{i\in\mathcal{L}_d} \left(\frac{1}{K_d(i)} \sum_{k\in\mathcal{K}_d(i)} \mathbf{1}\left\{s = s_d^*(i,k,\sigma)\right\}\right) (w_d(\sigma) + \Pi_d(\sigma))
\end{aligned}
$$

$$\implies \underbrace{\sum_{s\in\mathcal{M}_o} c_o(s,\sigma)y_o(s,\sigma)}_{\text{(1) Supply}} = \underbrace{\sum_d \alpha_d \sum_{b\in\mathcal{M}_d}\left(\frac{1}{K_d(b)}\sum_{k\in\mathcal{K}_d(b)} \mathbf{1}\left\{s_d^*(b,k,\sigma)\in\mathcal{M}_o\right\}\right) c_d(b,\sigma)y_d(b,\sigma)}_{\text{(2) Intermediate Input Demand}}$$

$$+ \underbrace{\sum_d \sum_{i\in\mathcal{L}_d}\left(\frac{1}{K_d(i)}\sum_{k\in\mathcal{K}_d(i)} \mathbf{1}\left\{s_d^*(i,k,\sigma)\in\mathcal{M}_o\right\}\right)(w_d(\sigma)+\Pi_d(\sigma))}_{\text{(3) Final Consumption Demand}}$$

We can simplify term (1) by making use of the labor market clearing condition as:

$$\begin{aligned}\text{Supply} &= \sum_{s\in\mathcal{M}_o} c_o(s,\sigma)y_o(s,\sigma)\\ &= \frac{w_o(\sigma)L_o}{1-\alpha_o}\end{aligned}$$

We can simplify term (2) as follows:

$$\begin{aligned}&\text{Intermediate Input Demand}\\ &= \sum_d \alpha_d \sum_{b\in\mathcal{M}_d}\left(\frac{1}{K_d(b)}\sum_{k\in\mathcal{K}_d(b)} \mathbf{1}\left\{s_d^*(b,k,\sigma)\in\mathcal{M}_o\right\}\right) c_d(b,\sigma)y_d(b,\sigma)\\ &= \sum_d \alpha_d \frac{\overbrace{\frac{1}{M_d}\sum_{b\in\mathcal{M}_d}\left(\frac{1}{K_d(b)}\sum_{k\in\mathcal{K}_d(b)} \mathbf{1}\left\{s_d^*(b,k,\sigma)\in\mathcal{M}_o\right\}\right) c_d(b,\sigma)y_d(b,\sigma)}^{(A)}}{\underbrace{\frac{1}{M_d}\sum_{b\in\mathcal{M}_d} c_d(b,\sigma)y_d(b,\sigma)}_{(B)}}\\ &\times \underbrace{\sum_{b\in\mathcal{M}_d} c_d(b,\sigma)y_d(b,\sigma)}_{=\frac{w_d(\sigma)L_d}{1-\alpha_d}}\end{aligned}$$

Term (A) can be simplified as follows:

$$
\begin{aligned}
(A) &= \frac{1}{M_d} \sum_{b \in \mathcal{M}_d} \left(\frac{1}{K_d(b)} \sum_{k \in \mathcal{K}_d(b)} \mathbf{1}\left\{ s_d^*(b,k,\sigma) \in \mathcal{M}_o \right\} \right) c_d(b,\sigma) y_d(b,\sigma) \\
&\xrightarrow{t \to \infty} \mathbb{E}\left[\left(\frac{1}{K_d(\cdot)} \sum_{k \in \mathcal{K}_d(\cdot)} \mathbf{1}\left\{ s_d^*(\cdot,k,\sigma) \in \mathcal{M}_o \right\} \right) c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right] \\
&= \mathbb{E}\left[\left(\frac{1}{K_d(\cdot)} \sum_{k \in \mathcal{K}_d(\cdot)} \mathbf{1}\left\{ s_d^*(\cdot,k,\sigma) \in \mathcal{M}_o \right\} \right) \right] \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\left(\frac{1}{K_d(\cdot)} \sum_{k \in \mathcal{K}_d(\cdot)} \mathbf{1}\left\{ s_d^*(\cdot,k,\sigma) \in \mathcal{M}_o \right\} \right) \mid K_d \right] \right] \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right] \\
&= \mathbb{E}\left[\frac{1}{K_d(\cdot)} \sum_{k \in \mathcal{K}_d(\cdot)} \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(\cdot,k,\sigma) \in \mathcal{M}_o \right\} \mid K_d \right] \right] \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right] \\
&= \mathbb{E}\left[\frac{1}{K_d(\cdot)} \sum_{k \in \mathcal{K}_d(\cdot)} \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(\cdot,\cdot,\sigma) \in \mathcal{M}_o \right\} \right] \right] \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right] \\
&= \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(\cdot,\cdot,\sigma) \in \mathcal{M}_o \right\} \right] \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right] \\
&= \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(\cdot,\cdot,\sigma) \in \mathcal{M}_o \right\} \right] \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right] \\
&= \pi_{od}\left(\bullet, -, \sigma_0 \right) \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right]
\end{aligned}
$$

Term (B) can be simplified as follows:

$$
\begin{aligned}
(B) &= \frac{1}{M_d} \sum_{b \in \mathcal{M}_d} c_d(b,\sigma) y_d(b,\sigma) \\
&\xrightarrow{t \to \infty} \mathbb{E}\left[c_d(\cdot,\sigma) y_d(\cdot,\sigma) \right]
\end{aligned}
$$

Substituting (A) and (B) back in the Intermediate Input Demand,we obtain:

$$
\text{Intermediate Input Demand} = \sum_d \alpha_d \pi_{od}\left(\bullet, -, \sigma_0 \right) \frac{w_d(\sigma) L_d}{1 - \alpha_d}
$$

We can simplify term (3) as follows:

Final Consumption Demand

$$
\begin{aligned}
&= \sum_d \sum_{i \in \mathcal{L}_d} \left(\frac{1}{K_d(i)} \sum_{k \in \mathcal{K}_d(i)} \mathbf{1}\left\{ s_d^*(i,k,\sigma) \in \mathcal{M}_o \right\} \right) w_d(\sigma) \\
&= \sum_d \left(\frac{1}{L_d} \sum_{i \in \mathcal{L}_d} \left(\frac{1}{K_d(i)} \sum_{k \in \mathcal{K}_d(i)} \mathbf{1}\left\{ s_d^*(i,k,\sigma) \in \mathcal{M}_o \right\} \right) \right) w_d(\sigma) L_d \\
&\xrightarrow{t\to\infty} \sum_d \mathbb{E}\left[\frac{1}{K_d(i)} \sum_{k \in \mathcal{K}_d(i)} \mathbf{1}\left\{ s_d^*(i,k,\sigma) \in \mathcal{M}_o \right\} \right] w_d(\sigma) L_d \\
&= \sum_d \mathbb{E}\left[\mathbb{E}\left[\frac{1}{K_d(i)} \sum_{k \in \mathcal{K}_d(i)} \mathbf{1}\left\{ s_d^*(i,k,\sigma) \in \mathcal{M}_o \right\} \mid N_d \right] \right] w_d(\sigma) L_d \\
&= \sum_d \mathbb{E}\left[\frac{1}{K_d(i)} \sum_{k \in \mathcal{K}_d(i)} \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(i,k,\sigma) \in \mathcal{M}_o \right\} \mid N_d \right] \right] w_d(\sigma) L_d \\
&= \sum_d \mathbb{E}\left[\frac{1}{K_d(i)} \sum_{k \in \mathcal{K}_d(i)} \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(i,k,\sigma) \in \mathcal{M}_o \right\} \right] \right] w_d(\sigma) L_d \\
&= \sum_d \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(\cdot,\cdot,\sigma) \in \mathcal{M}_o \right\} \right] w_d(\sigma) L_d \\
&= \sum_d \mathbb{E}\left[\mathbf{1}\left\{ s_d^*(\cdot,\cdot,\sigma) \in \mathcal{M}_o \right\} \right] w_d(\sigma) L_d \\
&= \sum_d \pi_{od}\left(\bullet, -, \sigma_0 \right) w_d(\sigma) L_d
\end{aligned}
$$

Putting these together we can further simplify the goods market clearing condition to obtain the desired result as follows:

$$
\begin{aligned}
\frac{w_o(\sigma) L_o}{1-\alpha_o} &= \sum_d \pi_{od}(\bullet, -, \sigma_0) \left(\frac{\alpha_d}{1-\alpha_d} + 1 \right) w_d(\sigma) L_d \\
\implies \quad \frac{w_o(\sigma) L_o}{1-\alpha_o} &= \sum_d \pi_{od}(\bullet, -, \sigma_0) \frac{w_d(\sigma) L_d}{1-\alpha_d}
\end{aligned}
$$

Since $\{w_d(\sigma)\}_d$ solves the above system of equations for a given realization of σ_0, irrespective of the realization of σ_1, we conclude that $w_d(\sigma) = w_d(\sigma_0)$. That is, $\{w_d : d \in \mathcal{J}\}$ solves the following system of equations for given realization of σ_0, irrespective to realization of σ_1.

$$
\frac{w_o L_o}{1-\alpha_o} = \sum_d \pi_{od}(\bullet, -) \frac{w_d L_d}{1-\alpha_d}
$$

D Appendix: Quantitative Analysis

D.1 Expected Utility & Welfare Changes

Households residing at location d are heterogeneous both in their numbers of needs and match-specific taste shocks of using different suppliers' goods to fulfill their needs. Welfare at any location is then calculated in expectation. That is, $V_d = \mathbb{E}\left[V_d\left(\cdot\right)\right]$. With Cobb-Douglas utilities across tasks, indirect utility of household i residing at d is given by:

$$V_d(i) = \frac{w_d}{\prod_{k \in \mathcal{K}_d(i)} p_d(i,k)^{1/K_d(i)}}$$

Expected indirect utility of households at location d can then be derived as:

$$\begin{aligned}
V_d &= \mathbb{E}\left[V_d\left(\cdot\right)\right] \\
&= \mathbb{E}\left[w_d \prod_{k \in \mathcal{K}_d(\cdot)} p_d(\cdot,k)^{-1/K_d(\cdot)}\right] \\
&= w_d \mathbb{E}\left[\mathbb{E}\left[\prod_{k \in \mathcal{K}_d(\cdot)} p_d(\cdot,k)^{-1/K_d(\cdot)} \mid K_d\right]\right] \\
&= w_d \mathbb{E}\left[\prod_{k \in \mathcal{K}_d(\cdot)} \mathbb{E}\left[p_d(\cdot,\cdot)^{-1/K_d(\cdot)} \mid K_d\right]\right] \\
&= w_d \mathbb{E}\left[\prod_{k \in \mathcal{K}_d(\cdot)} \Gamma\left(1 - \frac{1}{\zeta K_d\left(\cdot\right)}\right) A_d^{\frac{1}{\zeta K_d(\cdot)}}\right] \\
&= \mathbb{E}\left[\Gamma\left(1 - \frac{1}{\zeta K_d\left(\cdot\right)}\right)^{K_d(\cdot)}\right] w_d A_d^{\frac{1}{\zeta}}
\end{aligned}$$

Welfare changes, i.e., changes in expected indirect utility at location d in response to shocks can be calculated as:

$$\widehat{V}_d = \widehat{w}_d \widehat{A}_d^{1/\zeta},$$

where $\widehat{w}_d$ denotes the change in wage and $\widehat{A}_d$ denotes change in market access at d.